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Mathematics

ON q-LATTICES AND q-ALGEBRAS

A. M. GEVORGYAN*

Chair of Algebra and Geometry, YSU

In the present paper the concepts of q-lattice and q-algebra introduced in [1] and [2] are studied. The relationship between lattices (Boolean algebras) and q-lattices (q-algebras) is stated, and the results of [2] are improved.

***Keywords*:** q-lattices, q-algebras, on representation of q-algebras.

The concept of q-lattices and q-algebras. Let Q be a quasiorder (i.e. a reflexive and transitive binary relation) on set $A \neq \varnothing$. Then $E_Q = Q \cap Q^{-1} \subset A \times A$ is an equivalence relation, and $Q / E_Q \subset A / E_Q \times A / E_Q$ is a relation defined as $\langle B, C \rangle \in Q / E_Q$ for $B, C \in A / E_Q \leftrightarrow \langle b, c \rangle \in Q$ for each $\forall b \in B, \forall c \in C$, whereas $B, C \in A / E_Q$, is an order on A. For short, we write $\leq_Q$ instead of Q / E_Q, and denote by $[x]$ the equivalence class of $E_Q = Q \cap Q^{-1}$ containing the element $x \in A$.

Definition 1. The function $K : A / E_Q \to A$ is called a choice-function, if $K(B) \in B$ for any $B \in A / E_Q$. The triple (A, Q, K) is called an quasiordered set, if for any two classes $B, C \in A / E_Q$ there exist $\sup_{\leq_Q}(B, C)$ and $\inf_{\leq_Q}(B, C)$.

Definition 2. Algebra $(A, \vee, \wedge)$ is called a q-lattice, if for any $a, b, c \in A$ the operations $\vee, \wedge$ have the following properties:

1. $a \vee b = b \vee a$ and $a \wedge b = b \wedge a$ (commutativity);
2. $a \vee (b \vee c) = (a \vee b) \vee c$ and $a \wedge (b \wedge c) = (a \wedge b) \wedge c$ (associativity);
3. $a \vee (b \wedge a) = a \vee a$ and $a \wedge (b \vee a) = a \wedge a$ (weak absorption);
4. $a \vee (b \vee b) = a \vee b$ and $a \wedge (b \wedge b) = a \wedge b$ (weak idempotence);
5. $a \vee a = a \wedge a$ (equilibrium).

Lemma 1 (see [1]). Let (A, Q, K) be a quasiordered set. For each $x, y \in A$ we put $x \vee y = K\left(\sup_{\leq Q}([x],[y])\right)$ and $x \wedge y = K\left(\inf_{\leq Q}([x],[y])\right)$ then the algebra $(A, \vee, \wedge)$ is a q-lattice.

* E-mail: gev.ann84@mail.ru

Lemma 2 (see [1]). If $(A, \vee, \wedge)$ is a q-lattice then the relation $\langle a,b\rangle \in Q \leftrightarrow a\vee b = b\vee b$ is a quasiorder on set A, the mapping $K: A/E_Q \to A$ defined by $K([a]) = a\vee a$ is a choice-function, and the triple (A, Q, K) is a quasiordered set, where $\sup_{\leq Q}(B,C)$ and $\inf_{\leq Q}(B, C)$ are defined as

$$\sup_{\leq Q}(B,C) = [b\vee c] \text{ and } \inf_{\leq Q}(B,C) = [b\wedge c].$$

If the triple (A, Q, K) is a quasiordered set then $(A/E_Q, \sup_{\leq Q}, \inf_{\leq Q})$ is called an induced lattice.

Definition 3. A q-lattice $(A, \vee, \wedge)$ is called distributive, if it satisfies the distributivity condition $a\vee(b\wedge c) = (a\vee b)\wedge(a\vee c)$ for any $a,b,c\in A$.

Lemma 3. In any q-lattice $(A, \vee, \wedge)$ the following two properties of distributivity I. $a\vee(b\wedge c) = (a\vee b)\wedge(a\vee c)$ and II. $a\wedge(b\vee c) = (a\wedge b)\vee(a\wedge c)$ are equivalent for any $a,b,c\in A$.

Proof. Let us prove that II implies I.

$$(a\vee b)\wedge(a\vee c) = ((a\vee b)\wedge a)\vee((a\vee b)\wedge c) = (a\wedge a)\vee(c\wedge(a\vee b)) = (a\vee a)\vee(c\wedge(a\vee b)) =$$
$$= a\vee(c\wedge(a\vee b)) = a\vee((c\wedge a)\vee(c\wedge b)) = (a\vee(c\wedge a))\vee(c\wedge b) = (a\vee a)\vee(c\wedge b) = a\vee(c\wedge b).$$

The implication I→II is proved similarly.

Lemma 4. If $a\vee a' = a\vee a''$ and $a\wedge a' = a\wedge a''$ in a distributive q-lattice then $a'\vee a' = a''\vee a''$.

Proof.

$$a'\vee a' = a'\vee(a'\wedge a) = a'\vee(a''\wedge a) = (a'\vee a'')\wedge(a'\vee a) = (a'\vee a'')\wedge(a''\vee a) = a''\wedge(a'\vee a) =$$
$$= a''\wedge(a''\vee a) = a''\wedge a'' = a''\vee a''.$$

Definition 4. A q-lattice $(A, \vee, \wedge)$ is called limited, if there are elements 0 and 1 such that $0\wedge a = 0$ and $1\vee a = 1$ for any element $a\in A$.

Definition 5. A q-lattice $(A, \vee, \wedge)$ is called q-lattice with complements, if for any $a\in A$ there is $a'\in A$ such that $a'\wedge a = 0$ and $a'\vee a = 1$.

Lemma 5. If a' is a complement of a then a' is a complement for any element from the equivalence class containing a, and any element from the equivalence class containing a' is complement for a. In other words, if $a'\wedge a = 0$ and $a'\vee a = 1$, then $a'\wedge b = 0$ and $a'\vee b = 1$ $\forall b\in[a]$, $b'\wedge a = 0$ and $b'\vee a = 1$ $\forall b'\in[a']$.

Proof. Since the elements a and b belong to the same class, if and only if $a\vee a = b\vee b$ we obtain

$$a'\wedge b = a'\wedge(b\wedge b) = a'\wedge(a\wedge a) = a'\wedge a = 0,\ a'\vee b = a'\vee(b\vee b) = a'\vee)(a\vee a) = a'\vee a = 1,$$
$$b'\wedge a = (b'\wedge b')\wedge a = (a'\wedge a')\wedge a = a'\wedge a = 0,\ b'\vee a = (b'\vee b')\vee a = (a'\vee a')\vee a' = a'\vee a = 1.$$

Lemma 6. If a' and a'' are complements of a then $a'\vee a' = a''\vee a''$.

Proof. $a\vee a' = a\vee a'' = 1$ and $a\wedge a' = a\wedge a'' = 0 \to a'\vee a' = a''\vee a''$.

Definition 6. A distributive, limited q-lattice with complements is called a q-algebra.

Lemma 7. In any q-algebra $(A, \vee, \wedge)$ for any $a,b,x\in A$ there hold the following identities

1. $(a \vee b)' \vee x = a' \wedge b' \vee x,$ 3. $(a \vee b)' \wedge x = a' \wedge b' \wedge x,$ 5. $(a')' \vee x = a \vee x,$

2. $(a \wedge b)' \vee x = a' \vee b' \vee x,$ 4. $(a \wedge b)' \wedge x = a' \vee b' \wedge x,$ 6. $(a')' \wedge x = a \wedge x.$

Proof. Let us prove the first identity

$$(a \vee b) \vee (a' \wedge b') = a \vee (a' \wedge b') \vee b = (a \vee a') \wedge (a \vee b') \vee b = 1 \wedge (a \vee b') \vee b =$$
$$= (a \vee b') \vee b = a \vee (b \vee b') = a \vee 1 = 1,$$
$$(a \vee b) \wedge (a' \wedge b') = a \wedge (a' \wedge b') \vee b \wedge (a' \wedge b') = (a \wedge a') \wedge b' \vee (b \wedge b') \wedge a' =$$
$$= (0 \wedge b') \vee (0 \wedge a') = 0 \vee 0 = 0,$$

i.e. $a' \wedge b'$ is a complement for $a \vee b$, therefore, $(a \vee b)' \vee (a \vee b)' = = (a' \wedge b') \vee (a' \wedge b')$, that implies

$$(a \vee b)' \vee x = (a \vee b)' \vee (a \vee b)' \vee x = (a' \wedge b') \vee (a' \wedge b') \vee x = (a' \wedge b') \vee x .$$

Similarly we can prove the identities 2–4.

Let us prove the identity 5. $(a')'$ and a are complements of a', therefore,

$$(a')' \vee (a')' = a \vee a \Rightarrow (a')' \vee x = (a')' \vee (a')' \vee x = a \vee a \vee x = a \vee x .$$

The identity 6 is proved similarly to 5.

Definition 7. In q-algebra $(A, \vee, \wedge)$ a complement a' of an element a is called Boolean if it is idempotent, i.e. $a' \vee a' = a'$.

Lemma 8. Every element of q-algebra has a unique Boolean complement.

Proof. Let $(A, \vee, \wedge)$ be a q-algebra. We prove that if a' is a complement of $a \in A$ then the element $a^+ = a' \vee a'$ is its Boolean complement – $a \vee a^+ = a \vee (a' \vee a') = a \vee a' = 1,$ $a \wedge a^+ = a \wedge (a' \wedge a') = a \wedge a' = 0.$ Since the element $a^+ = a' \vee a'$ is idempotent, from Definition 7 we conclude that a^+ is a Boolean complement of a. Now we have to show its uniqueness. If both a' and a'' are Boolean complements of the element $a \in A$ then $a' = a' \vee a' = a'' \vee a'' = a''$.

Lemma 9. In any q-algebra $(A, \vee, \wedge)$ for Boolean complements there hold De Morgan's identities: $(a \vee b)^+ = a^+ \wedge b^+$, $(a \wedge b)^+ = a^+ \vee b^+$ and $1^+ = 0,\ 0^+ = 1,$ for every $a, b \in A$.

Proof. Let us check De Morgan's identities. Since $a^+ \wedge b^+$ is a complement of $(a \vee b)^+$ and $a^+ \wedge b^+$ is idempotent, from the definition of Boolean complement we obtain the first identity. The second identity is proved similarly. And since 1 and 0 are idempotent elements, then $1^+ = 0$ and $0^+ = 1$.

Lemma 10. If in q-algebra $(A, \vee, \wedge)$ the mapping $\varphi : a \to a^+$ is injective or surjective then $(A, \vee, \wedge, {}^+, 0, 1)$ is a Boolean algebra.

Proof. Let the mapping $\varphi : a \mapsto a^+$ be injective. We prove that algebra $(A, \vee, \wedge, {}^+, 0, 1)$ is Boolean. Assume the opposite, i.e. there exists an element $a \in A$ such that $a \vee a = b,\ a \neq b$. In this case

$$\left.\begin{array}{l} 1 = a \vee a^{+} = (a \vee a) \vee a^{+} = b \vee a^{+} \\ 0 = a \wedge a^{+} = (a \wedge a) \wedge a^{+} = b \wedge a^{+} \end{array}\right| \Rightarrow \varphi(a) = \varphi(b) \Rightarrow a = b ,$$

so we come to a contradiction.

If mapping $\varphi : a \to a^{+}$ is surjective then for any element $b \in A$ there exists an element $a \in A$ such that $\varphi(a) = b$ i.e. $a^{+} = b$, so $b \vee b = a^{+} \vee a^{+} = a^{+} = b \Rightarrow A$ is a Boolean algebra.

Corollary 1. If in q-algebra $(A, \vee, \wedge)$ the identity $(a^{+})^{+} = a$ holds for any element $a \in A$ then $(A, \vee, \wedge)$ is a Boolean algebra.

Lemma 11. If in q-algebra $(A, \vee, \wedge)$ any element $a \in A$ has only one complement then $(A, \vee, \wedge)$ is a Boolean algebra.

Proof. Assume the opposite, i.e. there exists an element $a \in A$ such that $a \vee a = b,\ a \neq b$. In this case

$$\left.\begin{array}{l} 1 = a \vee a' = \left(a \vee a\right) \vee a' = b \vee a' \\ 0 = a \wedge a' = \left(a \wedge a\right) \wedge a' = b \wedge a' \end{array}\right| \Rightarrow\ a' \in A \text{ has two complements} - a \text{ and } b.$$

So we come to a contradiction.

Theorem 1. Algebra $(A, \vee, \wedge)$ is a q-algebra, if and only if (iff) the induced lattice $\left(A / E_Q, \sup_{\leq Q}, \inf_{\leq Q}\right)$ is a Boolean algebra.

To prove the Theorem we first prove that:

- A q-lattice $(A, \vee, \wedge)$ is distributive, iff the induced lattice $\left(A / E_Q, \sup_{\leq Q}, \inf_{\leq Q}\right)$ is distributive.
- A q-lattice $(A, \vee, \wedge)$ is limited, iff the induced lattice $\left(A / E_Q, \sup_{\leq Q}, \inf_{\leq Q}\right)$ is limited.
- A q-lattice $(A, \vee, \wedge)$ is a q-lattice with complement, iff the induced lattice $\left(A / E_Q, \sup_{\leq Q}, \inf_{\leq Q}\right)$ is a q-lattice with complement.

The first statement is proved in [1]. Let us prove the second one. Let $(A, \vee, \wedge)$ be a limited q-lattice. Denote $O = [0]$ and $I = [0]$. In this case $\sup_{\leq Q}(A, I) = [a \vee 1] = [1] = I$, $\inf_{\leq Q}(A, O) = [a \wedge 0] = [0] = O$ for each class A / E_Q, and hence the induced lattice $\left(A / E_Q, \sup_{\leq Q}, \inf_{\leq Q}\right)$ is limited by $O = [0]$ and $I = [1]$. Now let the induced lattice $\left(A / E_Q, \sup_{\leq Q}, \inf_{\leq Q}\right)$ be limited, i.e. $\left(A / E_Q, \sup_{\leq Q}, \inf_{\leq Q}, I, O\right)$. Denote $K(O) = [0],\ K(I) = [1]$ then for each $a \in A$ hold

$$0 \wedge a = K(\inf_{\leq_Q}([0],[a])) = K(\inf_{\leq_Q}(O,[a])) = K(O) = 0,\ 1 \vee a = K(\sup_{\leq_Q}([1],[a])) =$$
$$= K(\sup_{\leq_Q}(I,[a])) = K(I) = 1$$

and, therefore, the q-lattice $(A, \vee, \wedge)$ is limited by 0 and 1. Now it remains to prove the third assertion. Let $(A, \vee, \wedge)$ be a q-lattice with complements. In this case the class $A' = [a'] \in A/E_Q$ (for each $a \in A$) is a complement of the class $A \in A/E_Q$. Indeed,

$$\sup_{\leq Q}(A, A') = \sup_{\leq Q}([a],[a']) = [a \vee a'] = [1] = I,$$
$$\inf_{\leq Q}(A, A') = \inf_{\leq Q}([a],[a']) = [a \wedge a'] = [0] = O.$$

Now, let the induced lattice $\left(A/E_Q, \sup_{\leq Q}, \inf_{\leq Q}\right)$ be q-lattice with complements. In this case every element of the class A' is a complement for any $a \in A$, since

$$a \vee a' = K(\sup_{\leq_Q}([a],[a'])) = K(\sup_{\leq_Q}(A, A')) = K(I) = 1,$$
$$a \wedge a' = K(\inf_{\leq_Q}([a],[a'])) = K(\inf_{\leq_Q}(A, A')) = K(O) = 0.$$

Example 1. Let us give an example of a non-Boolean three-element q-algebra. We take the set $A = \{0,1,a\}$ and give the operations $\vee, \wedge, '$ by Tables 1, 2 and 3. The graphs of the q-algebra and the induced Boolean algebra are illustrated in Fig. 1 and 2, respectively.

Table 1

$\vee$	1	0	a
1	1	1	1
0	1	0	1
a	1	1	1

Table 2

$\wedge$	1	0	a
1	1	0	0
0	0	0	0
a	0	0	1

Table 3

x	1	0	a
x'	0	1,a	0

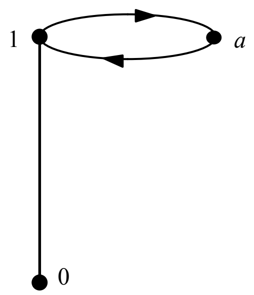

Fig. 1.

Fig. 2.

On representation of *q*-algebras.

Definition 8. The set of binary functions $G(A)$, $f: A^2 \to A$ $(A \neq 0)$ is called a set of g-functions, if it satisfies the following four properties for all $f, g, h \in G$:

1. $f(x, x) = g(x, x)$ (uniformity);

2. $\langle x,y\rangle \neq \langle u,v\rangle \Rightarrow f\left(f(x,y), f(u,v)\right) = f(x,v)$ (conditional diagonality);

3. $\langle x,u\rangle \neq \langle y,v\rangle \Rightarrow f\left(g(x,y), g(u,v)\right) = g\left(f(x,u), f(y,v)\right)$ for $\langle x,y\rangle \neq \langle u,v\rangle$ (conditional commutativity);

4. $f \neq pr_2 \to f(x,y) = f\left(x, g(y,y)\right)$, $f \neq pr_1 \to f(x,y) = f\left(h(x,x), y\right)$, $pr_1 \neq f \neq pr_2 \to f(x,y) = f\left(h(x,x), g(y,y)\right)$, where $pr_i(x_1,x_2) = x_i$, i=1,2 [3] (conditional absorption).

Let $(A, \vee, \wedge)$ be a q-algebra. Denote by $G(A)$ the set of all binary functions over A defined as

$$f_b(x,y)=\begin{cases} b \quad if \quad \langle x,y\rangle=\langle 1,0\rangle, \\ (x\wedge b)\vee(y\wedge b') \quad if \quad \langle x,y\rangle\neq\langle 1,0\rangle. \end{cases}$$

Note that if b' and b'' are complements of b then

$$y\wedge b'=y\wedge(b'\wedge b')=y\wedge(b''\wedge b'')=y\wedge b''.$$

Theorem 2. Let A be a q-algebra. The set of binary functions $G(A)$ is a set of g-functions, if A is a Boolean algebra.

Proof. If $G(A)$ is a set of g-functions then, due to condition of diagonality, in the case $u\neq 1$ $f_b(f_b(1,0),f_b(u,0))=f_b(1,0)=b$ holds for any element $b\in A$; on the other hand,

$$f_b\left(f_b\left(1,0\right),f_b\left(u,0\right)\right)=f_b\left(b,f_b\left(u,0\right)\right)=f_b\left(b,\left(b\wedge u\right)\vee\left(b'\wedge 0\right)\right)=f_b\left(b,b\wedge u\right)=$$
$$\left(b\wedge b\right)\vee\left(b\wedge u\wedge b'\right)=b\vee\left(u\wedge 0\right)=b\vee 0=b\vee b.$$

Therefore, $b=b\vee b$ for any $b\in A$ and thus A is a Boolean algebra. Now, note that if A is a Boolean algebra then $f_b(1,0)==(1\wedge b)\vee(0\wedge b')=b\vee 0=b$, so $f_b(x,y)$ is defined as follows: $f_b(x,y)=(x\wedge b)\vee(y\wedge b')$.

Let us prove that set of binary function $G(A)$ satisfies the following conditions:

1') $f(x,x)=x;$

2') $f(f(x,y),f(u,v))=f(x,v);$

3') $f(g(x,y),g(u,v))=g(f(x,u),f(y,v)),$

from which follow the properties 1–4. Thus,

$$f_b(x,x)=(x\wedge b)\vee(x\wedge b')=x\wedge(b\vee b')=x\wedge 1=x,$$
$$f_b(f_b(x,y),f_b(u,v))=(((x\wedge b)\vee(y\wedge b'))\wedge b)\vee(((u\wedge b)\vee(v\wedge b'))\wedge b')=$$
$$=(x\wedge b)\vee(v\wedge b')=f_b(x,v),$$
$$f_b(f_c(x,y),f_c(u,v))=(((x\wedge c)\vee(y\wedge c'))\wedge b)\vee(((u\wedge c)\vee(v\wedge c'))\wedge c')=$$
$$=(((x\wedge b)\vee(u\wedge b'))\wedge c)\vee(((y\wedge b)\vee(v\wedge b'))\wedge c')=f_c(f_b(x,y),f_b(u,v)).$$

Let $G(A)$ be a set of g-functions. Define operation $\vee, \wedge, ', o, j$ as follows:

$$(f\vee g)(x,y)=f(x,g(x,y)),$$
$$(f\wedge g)(x,y)=f(g(x,y),y),$$
$$f'(x,y)=f(y,x),$$
$$o(x,y)=f(y,y),\ \ j(x,y)=f(x,x).$$

Theorem 3. $A(G(A))=(G(A),\vee,\wedge,^{/},o(x,y),j(x,y))$ is a Boolean algebra.

Corollary 2. The q-algebra $A(G)=(G,\vee,\wedge,^{+},o,j)$ is isomorphic to the algebra $A(G(A))=(G(A),\vee,\wedge,^{/},f_0(x,y),f_1(x,y))$, if and only if $A(G)=(G,\vee,\wedge,^{+},o,j)$ is a Boolean algebra and isomorphism is given as follows

$$b\mapsto f_b(x,y)=(x\wedge b)\vee(y\wedge b^{+}).$$

Operations $\vee, \wedge, '$ in algebra $A(G(A))$ are defined as

$$(f_b \vee f_c)(x,y) = f_b(x, f_c(x,y)),\ (f_b \wedge f_c)(x,y) = f_b(f_c(x,y),y),\ f_b'(x,y) = f_b(y,x).$$

Proof. If the q-algebra $A(G) = (G, \vee, \wedge, ^+, o, j)$ is isomorphic to the algebra $A(G(A)) = (G(A), \vee, \wedge, ^/, f_0(x,y), f_1(x,y))$ then $f_{b\vee c}(x,y) = (f_b \vee f_c)(x,y)$, and particularly

$$\left.\begin{array}{l} f_{b\vee 0}(1,0) = (f_b \vee f_0)(1,0), \\ f_{b\vee 0}(1,0) = b \vee 0 = b \vee b, \\ (f_b \vee f_0)(1,0) = f_b(1, f_0(1,0)) = f_b(1, f_0(1,0)) = f_b(1,0) = b, \end{array}\right\} \Rightarrow b \vee b = b$$

for any $b \in A$.

Therefore, $A(G) = (G, \vee, \wedge, ^+, o, j)$ is a Boolean algebra. Now, if $A(G) = (G, \vee, \wedge, ^+, o, j)$ is a Boolean algebra then, due to the Theorem 2, $G(A)$ is a set of g-functions, since $f_0(x,y) = (x \wedge 0) \vee (y \wedge 1) = y = f(y,y) = O(x,y)$. Similarly we can prove that $f_1(x,y) = j(x,y)$. So, due to the Theorem 3, $A(G(A))$ is a Boolean algebra. Thus it remains to prove isomorphism of these Boolean algebras:

$$f_{b\vee c}(x,y) = (x \wedge (b \vee c)) \vee (y \wedge (b \vee c)') = (x \wedge b) \vee (x \wedge c) \vee (y \wedge b' \wedge c') = (x \wedge b) \vee$$
$$\vee (x \wedge c) \wedge (b \vee b') \vee (y \wedge c') \wedge b' = (x \wedge b) \vee ((x \wedge c) \wedge b) \vee ((x \wedge c) \wedge b') \vee (y \wedge c') \wedge$$
$$\wedge b' = (x \wedge b) \vee ((x \wedge b) \wedge c) \vee ((x \wedge c) \wedge b') \vee (y \wedge c') \wedge b' = (x \wedge b) \vee ((x \wedge c) \wedge b') \vee$$
$$\vee ((y \wedge c') \wedge b') = (x \wedge b) \vee ((x \wedge c) \vee (y \wedge c')) \wedge b' = f_b(x, f_c(x,y)) = (f_b \vee f_c)(x,y).$$

Analogously it is proved that $f_{b\wedge c}(x,y) = (f_b \wedge f_c)(x,y)$.

$$f_{b'}(x,y) = (x \wedge b') \vee (y \wedge (b')') = (x \wedge b') \vee (y \wedge b) = f_b(y,x) = f_b'(x,y)..$$

The last two theorems and the corollary improve the results of [2].

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Ա.Մ. Գևորգյան

q-Կավարների և q-հանրահաշիվների մասին

Աշխատանքում հետազոտվում են q-կավարները և q-հանրահաշիվները: Ուսումնասիրվում է փոխմիարժեք համապատասխանությունը կավարների (բուլյան հանրահաշիվների) և q-կավարների (q-հանրահաշիվների) միջև: Ճշտվում են նաև [2] հոդվածի արդյունքները:

А. М. Геворкян.

О *q*-решетках и *q*-алгебрах.

В настоящей работе изучены понятия *q*-решетки и *q*-алгебры, введенные в работах [1] и [2]. Устанавливается взаимосвязь между решетками (булевыми алгебрами) и *q*-решетками (*q*-алгебрами), а также уточняются результаты статьи [2].