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Mathematics

ON FACTORIZATION OF A CLASS OF SECOND ORDER MATRIX-FUNCTIONS

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The paper suggests a factorization construction method for a class of second order matrix-functions. The left lower element of these matrix-functions may be represented as combinations of other three elements of matrix-function and two functions, meromorphic respectively in interior and exterior regions of the unit disk.

Keywords**:** factorization, matrix-function, partial indices.

1°. We denote by W the Wiener algebra of continuous functions defined on the unit circle Γ and representable as absolutely convergent Fourier series. By $W^{n\times m}$ we denote the set of all those matrix-functions (MF) of order $n\times m$, the components of which belong to W. For short we use standard notations W^n and W instead of $W^{n\times 1}$ and $W^{1\times 1}$.

Let $\langle f\rangle_k = \dfrac{1}{2\pi i}\int\limits_{\Gamma} f(z)\, z^{-k-1}\, dz$ be the Fourier coefficients of $f\in W^{n\times m}$. We define by equalities

$$(\mathcal{P}_j^- f)(t)=\sum_{k=-\infty}^{j-1}\langle f\rangle_k\, t^k\,,\quad (\mathcal{P}_j^+ f)(t)=\sum_{k=j}^{+\infty}\langle f\rangle_k\, t^k$$

projections $\mathcal{P}_j^{\pm}$ ($j\in Z$) acting in $W^{n\times m}$.

Consider the following spaces: $W_+^{n\times m}=\operatorname{Im}\mathcal{P}_0^+$, $W_-^{n\times m}=\operatorname{Im}\mathcal{P}_1^-$, $\overset{\circ}{W}{}_-^{n\times m}=\operatorname{Im}\mathcal{P}_0^-$. Under factorization of a MF $G\in W^{2\times 2}$ in W we understand its representation in the form $G=G_-\Lambda G_+$, where G_-, $G_-^{-1}\in W_-^{2\times 2}$, G_+, $G_+^{-1}\in W_+^{2\times 2}$, $\Lambda(t)=\operatorname{diag}\left[t^{\varkappa_1},t^{\varkappa_2}\right]$, $t\in\Gamma$, χ_1, $\chi_2\in Z$ and $\chi_1\le\chi_2$. The numbers χ_1,χ_2 are called partial indices of the MF G. Constructive methods for building factorizations are known only for narrow classes of matrix-functions (see, e.g., [1–6]).

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Let $G_{11}, G_{12}, G_{22}, h_1, h_2 \in W$. We assume that there exist some polynomials q_- and q_+ ($\deg q_- = \nu_+$, $\deg q_+ = n_-$), whose roots lie correspondingly in Γ_+ and Γ_-, and there exists a non-negative integer N ($N \geq n_-$), such that $p_- = \dfrac{q_+ h_1}{\tau^N} \in W_-$, $p_+ = q_- h_2 \in W_+$, where $\tau(t) = t$ $(t \in \Gamma)$.

A class of second order matrix-functions, defined on the unit circle $\Gamma = \{z \in \mathbb{C};\ |z| = 1\}$ of a form

$$G = \begin{pmatrix} G_{11} & G_{12} \\ h_1 G_{11} + h_2 G_{22} - h_1 h_2 G_{12} & G_{22} \end{pmatrix} \tag{1}$$

under assumption that h_1, h_2 are meromorphic functions correspondingly in domains $\Gamma_- = \{z \in \mathbb{C};\ |z| > 1\}$ and $\Gamma_+ = \{z \in \mathbb{C};\ |z| < 1\}$, is considered in [7]. In [7] we obtained some explicit formulas for the partial indices of MF of the form (1). In the present paper for this class of matrix-functions we suggest a constructive method of recovering factors $G_\pm$. The matrix-function factorization problem (1) is reduced to a successive factorization of two scalar functions and to the solution of explicit finite systems of linear equations.

2°. We assume below that

$$q_-(0) \neq 0. \tag{2}$$

This assumption is not essential by the following reason. If $q_-(0) = 0$ and $\alpha \in \Gamma_+$ are such that $q_-(\alpha) \neq 0$, then it is not difficult to verify, that MF $G_\zeta = G \circ \zeta$, where $\zeta(z) = (z + \alpha)/(1 + \bar{\alpha} z)$, also has structure of (1), if one replaces q_- with $\hat{q}_-(t) = (1 + \bar{\alpha} t)^{\nu_+} q_-(\zeta(t))$. One can easily see, that $q_-(0) = q_-(\alpha) \neq 0$. Besides, since ζ maps Γ_+ into Γ_+, then factorization of MF G in a simple way is restored by factorization of G_ζ.

3°. We consider functions

$$V_1 = \tau^N (q_- G_{11} - p_+ G_{12})/(q_+ q_-), \quad V_2 = \tau^N (\tau^{-N} q_+ G_{22} - p_- G_{12})/(q_+ q_-). \tag{3}$$

Further, we assume that the following conditions are satisfied

$$V_i(t) \neq 0 \quad (t \in \Gamma). \tag{4}$$

Factorization of MF G exists, if and only if conditions (4) are satisfied (see [7]).

Assume that $\chi_i = \operatorname{ind} V_i = \dfrac{1}{2\pi} \operatorname{var} \arg V_i(t) \Big|_{t \in \Gamma}$, $i = 1, 2$, $\chi_0 = \max\{\chi_1, \chi_2\}$, $\nu_- = N_+(\chi_1 - \chi_2)$,

$$V_i^\pm = \exp \mathcal{P}_0^\pm (\ln(t^{-\chi_i} V_i)), \quad i = 1, 2, \quad V_{12}^\pm = V_2^\pm \exp \mathcal{P}_0^\pm \left(\frac{\tau^{N-\chi_1} G_{12}}{V_1^- V_2^+ q_+ q_-} \right), \tag{5}$$

$$V_\pm = \begin{pmatrix} V_1^\pm & V_{12}^\pm \\ 0 & V_2^\pm \end{pmatrix}. \tag{6}$$

The MF $G_0 = \tau^{-\chi_0} G$ allows representation $G_0 = AB$ (see [7]), where

$$A = \begin{pmatrix} \frac{q_+}{t^N} & 0 \\ p_- & 1 \end{pmatrix} \begin{pmatrix} t^{\chi_1-\chi_0} & 0 \\ 0 & t^{\chi_2-\chi_0} \end{pmatrix} V_-, \quad B = V_+ \begin{pmatrix} 1 & 0 \\ p_+ & q_- \end{pmatrix}.$$

For $j \in Z$ we denote $\hat{j} = \frac{1}{2}(j + |j|)$, $\check{j} = \frac{1}{2}(j - |j|)$. We define also a MF $U = \tau^{-\nu_-} P_{\nu_-}^-(B^{-1}) P_{\nu_-}^+(A^{-1})$ and $\mathcal{B}_j$, $\mathcal{A}_j$ ($j > -\nu_-$, $\hat{j} + \nu_- > 1$), $\mathcal{U}_j$, $\mathcal{K}_j$ ($j > -\nu_-$) by following formulas:

$$\mathcal{B}_j = \begin{pmatrix} \langle B^{-1} \rangle_{-1-\check{j}} & \langle B^{-1} \rangle_{-2-\check{j}} & \cdots & \langle B^{-1} \rangle_{-\nu_- - j+1} \\ \langle B^{-1} \rangle_{-2-\check{j}} & \langle B^{-1} \rangle_{-3-\check{j}} & \cdots & \langle B^{-1} \rangle_{-\nu_- - j} \\ \cdots & \cdots & \cdots & \cdots \\ \langle B^{-1} \rangle_{-\nu_+} & \langle B^{-1} \rangle_{-\nu_+-1} & \cdots & \langle B^{-1} \rangle_{-(\nu_++\nu_-)+2-\hat{j}} \end{pmatrix}, \tag{7}$$

$$\mathcal{A}_j = \begin{pmatrix} 0 & \langle A^{-1} \rangle_{\nu_--1} & \langle A^{-1} \rangle_{\nu_--2} & \cdots & \langle A^{-1} \rangle_{1-\hat{j}} \\ 0 & 0 & \langle A^{-1} \rangle_{\nu_--1} & \cdots & \langle A^{-1} \rangle_{2-\hat{j}} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & \langle A^{-1} \rangle_{\nu_--1} \end{pmatrix}, \tag{8}$$

$$\mathcal{U}_j = \begin{pmatrix} \langle U \rangle_{-1-\check{j}} & \langle U \rangle_{-2-\check{j}} & \cdots & \langle U \rangle_{-\nu_- - j} \\ \langle U \rangle_{-2-\check{j}} & \langle U \rangle_{-3-\check{j}} & \cdots & \langle U \rangle_{-\nu_- - j-1} \\ \cdots & \cdots & \cdots & \cdots \\ \langle U \rangle_{-\nu_+} & \langle U \rangle_{-\nu_+-1} & \cdots & \langle U \rangle_{-(\nu_++\nu_-)+1-\hat{j}} \end{pmatrix}, \tag{9}$$

$$\mathcal{K}_j = \mathcal{U}_j + \mathcal{B}_j \mathcal{A}_j \text{ for } \nu_- + \hat{j} > 1, \; \mathcal{K}_j = \mathcal{U}_j \text{ for } \nu_- + \hat{j} = 1. \tag{10}$$

Results of [7] imply, that partial indices of MF G_0 are equal to $-\nu_- + \nu_0$ and $\nu_+ - \nu_0$, where $\nu_0 = \operatorname{card}\{j\,;\; r_j = 2\theta_j + r_{j-1},\, j = -\nu_- + 1, \ldots, \nu_+\}$. Here $r_{-\nu_-} = \nu_-$, $r_j = \operatorname{rang} \mathcal{K}_j$ for $j > -\nu_-$ (particularly, $r_{\nu_+} = \nu_+$), $\theta_j = 1$ when $j > 0$, and $\theta_j = 0$ when $j < 0$, $j \in Z$. We denote $\xi_1 = -\nu_- + \nu_0 + 1$, $\xi_2 = \nu_+ - \nu_0 + 1$.

4°. We denote by $\mathcal{L}_j$ ($j < 0$) the space of vector-polynomials of z^{-1} of the form $\sum\limits_{k=j}^{-1} \varphi_k z^k$, where $\varphi_k \in \mathbb{C}^2$. For $j = 0$ we assume that $\mathcal{L}_0 = \{0\}$. Let $\mathbb{C}^{2n} = \mathbb{C}^2 \times \mathbb{C}^2 \times \cdots \times \mathbb{C}^2$. We define operators $\Psi_j : \mathbb{C}^{2(\nu_- + \hat{j})} \to \mathcal{L}_{-(\nu_- + \hat{j})}$, $\Psi_j q = \sum\limits_{k=-(\nu_-+\hat{j})}^{-1} q_k t^k$, where $q = \left[q_{-(\nu_-+\hat{j})} \cdots q_{-1} \right]$, $q_k \in \mathbb{C}^2$ ($k = -(\nu_- + \hat{j}), \ldots, -1$).

We define the families of Hankel operators $H_j^- : \overset{\circ}{W}{}_-^2 \to W_+^2$, $H_j^+ : W_+^2 \to \overset{\circ}{W}{}_-^2$ ($j \in Z$) and Toeplitz operators $T_j : W_+^2 \to W_+^2$ ($j \in Z$) by formula

$$H_j^-\varphi = \mathcal{P}_0^+\left(\tau^{\hat{j}}(A^{-1}\varphi)\right),\ H_j^+\varphi = \mathcal{P}_0^-\left(\tau^{\check{j}}(B^{-1}\varphi)\right),\ T_j\varphi = \mathcal{P}_0^+\left(\tau^{-j}G_0\varphi\right).$$

Reasoning by analogy to the proof of Proposition 3 in [7], one can verify that for $j > -\nu_-$ the kernels of finite-dimensional operators $\mathcal{K}_j = H_j^+ H_j^-\big|_{\mathcal{L}_{-(\nu_-+\hat{j})}}$ are connected with kernels $\mathcal{K}_j$ by relations: $\ker\mathcal{K}_j = \Psi_j \ker\mathcal{K}_j$. The Lemma in [7] implies

$$N_j = \ker T_j = \left\{\tau^{\check{j}}B^{-1}\mathcal{P}_0^+\left(\tau^{\hat{j}}A^{-1}\Psi_j q\right); q \in \ker\mathcal{K}_j\right\},\quad j > -\nu_-. \tag{11}$$

The Toeplitz operators T_j may be defined (by the same formula) on wider spaces $L_2^+ = \mathcal{P}_0^+(L_2(\Gamma))$. Note that in this case kernels $N_j = \ker T_j$ are not extendable. Indeed, since $G_0 \in W^{2\times 2}$, then its any factorization in L_2 is a factorization in W (see [2]). And since any solution $T_j\varphi = 0$ in L_2^+ has the form $G_+^{-1}p$, where p is a polynomial, then φ belongs also to W_+^2.

Along with the spaces N_j ($j \in Z$) we consider also hereditary spaces $\widehat{N}_j = N_j + \tau N_j = \{\varphi + \tau\psi;\ \varphi,\psi \in N_j\}$. Taking into account the abovementioned remark, it is easy to see that $\widehat{N}_j \subset N_{j+1}$, $j \in Z$. The direct complement M_j of the space $\widehat{N}_j$ in N_{j+1} is called $(2,j)_+$-index subspace of MF G_0. It is known (see [8]) that $N_j = \{0\}$ for $j \le \xi_1 - 1$, and $N_j = \widehat{N}_{j-1}$ for all $j \in Z \setminus \{\xi_1, \xi_2\}$. Besides, the following statement is true.

Proposition 1. If $\xi_1 \ne \xi_2$, then spaces M_{ξ_1-1}, M_{ξ_2-1} are one-dimensional, and if $\xi_1 = \xi_2$, then the unique nonzero $(2,j)_+$-index subspace M_{ξ_1-1} is two-dimensional.

Proof. Since MF G_0 allows factorization in the sense of the Section 1°, then it allows factorization in the space L_2 (see [5]), and, therefore, it has a finite $(r_+,2)$-indexation (see [9], Theorem 4). Besides, the $(r_+,2)$-partial indices coincide with $\xi_1 - 1$, $\xi_2 - 1$. Consequently, due to Theorem 5 in [8], $\dim M_{\xi_1-1} + \dim M_{\xi_2-1} = 2$, if $\xi_1 \ne \xi_2$, and $\dim M_{\xi_1-1} = 2$, if $\xi_1 = \xi_2$. Taking into account that $\dim M_{\xi_i-1}$ ($i = 1,2$) are nonzero spaces (see [9]), we complete the proof of Proposition 1.

We choose vector-functions (VF) φ_1 and φ_2 as follows. Assume, that for $\xi_1 \ne \xi_2$ φ_1 and φ_2 are bases for index subspaces M_{ξ_1-1} and M_{ξ_2-1} respectively, and VFs φ_1 and φ_2 are a basis of subspace M_{ξ_1-1} when $\xi_1 = \xi_2$.

We define $G_+^{-1}=[\varphi_1,\varphi_2]$ (i.e. columns of MF G_+^{-1} are formed with VFs φ_1 and φ_2), $\Lambda_0=\operatorname{diag}\left[t^{\xi_1-1},t^{\xi_2-1}\right]$ and $G_-=G_0G_+^{-1}\Lambda_0^{-1}$, then the following statement is true.

Proposition 2. The representation $G_0=G_-\Lambda_0G_+$ is a factorization of MF G_0.

Proof. G_0 has a finite $(r_+,2)$-indexation with $(r_+,2)$-partial indices ξ_1-1, ξ_2-1, and, therefore, due to Theorem 2 from [9], the representation $G_0=G_-\Lambda_0G_+$ is a $(r_+,2)$-index factorization. It follows from Theorem 4 [9], that G_0 is a factorization in L_2 as well. Since $G\in W^{2\times2}$, then a factorization in L_2 coincides with factorization in W, i.e. with factorization in the sense of the Section 1° (see [2]). The proof is thus completed.

5°. We introduce the spaces $N_j(0)=\{\varphi(0)\,;\varphi\in N_j\}$, $\widehat{N}_j(0)=\{\varphi(0);\varphi\in\widehat{N}_j\}$ and $M_j(0)=\{\varphi(0)\,;\ \varphi\in M_j\}$. Obviously, $\widehat{N}_j(0)=N_j(0)$, $N_j(0)=\{0\}$ when $j\le\xi_1-1$, and $M_j(0)=\{0\}$ when $j\notin\{\xi_1-1,\xi_2-1\}$. Let $\xi_1\ne\xi_2$. Since $\det G_+^{-1}(0)\neq0$, then Propositions 1 and 2 imply $\dim M_{\xi_1-1}(0)=\dim M_{\xi_2-1}(0)=1$ and $M_{\xi_1-1}(0)\dotplus M_{\xi_2-1}(0)=\mathbb{C}^2$. Hence, taking into account the equalities $N_j(0)\cap M_j(0)=\{0\}$, $N_{j+1}(0)=N_j(0)\dotplus M_j(0)$, $j\in Z$ (see [8]), we get $N_{\xi_1}(0)=\cdots=N_{\xi_2-1}(0)=M_{\xi_1-1}(0)$ and $N_j=\mathbb{C}^2$ for $j\ge\xi_2$. For $\xi_1=\xi_2$ we have $\dim M_{\xi_1-1}(0)=2$, and for $j\ge\xi_2$ we get $N_j=\mathbb{C}^2$.

6°. For $j\in Z$, satisfying the inequality $\nu_-+\hat{j}\ge1$, we define matrices $\mathcal{K}_j'$ by formula $\mathcal{K}_j'=\left[\left\langle A^{-1}\right\rangle_{\nu_--\check{j}}\cdots\left\langle A^{-1}\right\rangle_{1-j}\right]$. Vectors $\overline{q}_1\in\mathbb{C}^{2(\nu_-+\hat{\xi}_1)}$, $\overline{q}_2\in\mathbb{C}^{2(\nu_-+\hat{\xi}_2)}$ are called a factorization pair for the MF G, if $\overline{q}_i\in\ker\mathcal{K}_{\xi_i}$ ($i=1,2$), and vectors $\mathcal{K}_{\xi_1}'\overline{q}_1$, $\mathcal{K}_{\xi_2}'\overline{q}_2$ are nonzero and linearly independent. Obviously, the existence of factorization pair $\overline{q}_1$, $\overline{q}_2$ is equivalent to the existence of nonzero linearly independent vectors $y_1,y_2\in\mathbb{C}^2$, such that

$$\begin{pmatrix}\mathcal{K}_{\xi_i} & 0\\ \mathcal{K}_{\xi_i}' & -E_2\end{pmatrix}\begin{pmatrix}\overline{q}_i\\ \overline{y}_i\end{pmatrix}=0,\qquad i=1,2, \tag{12}$$

where E_2 is a unity matrix.

Proposition 3. MF G possesses a factorization pair.

Proof. Assume that $\nu_-+\hat{j}>0$ and $\varphi\in N_j$. Due to (11) there exists $\overline{q}=\left[q_{-(\nu_-+\hat{j})}\cdots q_{-1}\right]\in\ker\mathcal{K}_j$ ($q_s\in\mathbb{C}^2$, $s=-(\nu_-+\hat{j}),\dots,j-1$), such that $\varphi(t)=t^{\check{j}}\,B^{-1}(t)\mathcal{P}_0^+\left(t^{\hat{j}}A^{-1}\Psi_jq\right)$. Using equality $(\Psi_jq)(t)=\sum\limits_{k=-(\nu_-+\hat{j})}^{-1}q_kt^k$, one can verify that

$$\varphi(t)=B^{-1}(t)\sum_{m=0}^{\infty}\left(\sum_{k=-(\nu_-+\hat{j})}^{-1}\left\langle A^{-1}\right\rangle_{m-k-\hat{j}}q_k\right)t^{m+\breve{j}}. \tag{13}$$

$B(0)$ is an invertible matrix, and hence $N_j(0)=\{B^{-1}(0)\mathcal{K}_j'\overline{q}\ ;\ \overline{q}\in\ker\mathcal{K}_j\}$. Now the existence of the factorization pair follows from the properties of spaces $N_j(0)$ (see Section 5°). Proposition is thus proved. Note also that in the neighborhood of $z=0$ the function $B(z)\varphi(z)$ is analytical, and, therefore,

$$\sum_{k=-(\nu_-+\hat{j})}^{-1}\left\langle A^{-1}\right\rangle_{m-k-\hat{j}}q_k=0,\qquad m=0,1,\dots,\breve{j}-1. \tag{14}$$

7°. The main result of the present paper is the following

Theorem. Let $\overline{q}_1$, $\overline{q}_2$ be a factorization pair for G, $\chi_i=\xi_i+\chi_0$,

$$\varphi_i=t^{\breve{\xi}_i}B^{-1}(t)H_{\xi_i}^{-}\Psi_{\xi_i}\overline{q}_i,\ \ i=1,2, \tag{15}$$

$$G_+^{-1}=[\varphi_1,\varphi_2],\ \Lambda(t)=\operatorname{diag}[t^{\chi_1},t^{\chi_2}],\ G_-=GG_+^{-1}\Lambda^{-1}.$$

Then the representation $G=G_-\Lambda G_+$ is a factorization of MF G.

Proof. It follows from the definition of factorization pair that $\mathcal{K}_{\xi_i}q_i=0$ ($i=1,2$). Due to (11), VFs φ_j, defined by the equality (15), belong to spaces N_{ξ_i}.

We consider first the case $\xi_1=\xi_2$. Since $\mathcal{K}_{\xi_1}'\overline{q}_1$ and $\mathcal{K}_{\xi_2}'\overline{q}_2$ are linearly independent and $\det B(0)\neq 0$, then $\varphi_1(0)$ and $\varphi_2(0)$ are linearly independent as well. Therefore, VFs φ_1, φ_2 are also linearly independent. We have $N_{\xi_1}=M_{\xi_1-1}$, and hence φ_1, φ_2 form a basis of M_{ξ_1-1}, and the proof of Theorem follows from Proposition 2.

Assume now $\xi_1\neq\xi_2$. $\varphi_1(0)\neq 0$, i.e. $\varphi_1\neq 0$, since $\det B(0)\neq 0$ and $\mathcal{K}_{\xi_1}'\overline{q}_1\neq 0$. Further, $M_{\xi_1-1}=N_{\xi_1}$ is an one-dimensional space, and, therefore, φ_1 is the basis of this space. Since $\mathcal{K}_{\xi_1}'\overline{q}_1$ and $\mathcal{K}_{\xi_2}'\overline{q}_2$ are linearly independent, then vectors $\varphi_1(0)$, $\varphi_2(0)$ are also linearly independent. Taking into account that $N_{\xi_1-1}(0)=N_{\xi_1}(0)=\operatorname{span}\{\varphi_1(0)\}$, we obtain that $\varphi_2(0)\notin N_{\xi_2-1}(0)=\widehat{N}_{\xi_2}(0)$. Thus, $\varphi_2\in N_{\xi_2}$, but $\varphi_2\notin\widehat{N}_{\xi_2-1}$, i.e. φ_2 belongs to some subspace M_{ξ_2-1}. It remains to apply the Proposition 2.

Theorem is thus proved.

Basing on the Theorem proved above we suggest the following scheme of constructing a factorization of the MF G:

1. Constructing MF $V_\pm$ according to (5), (6).
2. Constructing matrices $\mathcal{K}_j$ ($j=-\nu_-+1,\dots,\nu_+$) according to (7)–(10).
3. Determining numbers $r_j=\operatorname{rang}\mathcal{K}_j$ ($j=-\nu_-+1,\dots,\nu_+$), $r_{-\nu_-}=\nu_-$, ν_0, ξ_i, χ_i ($i=1,2$).
4. Constructing a factorization pair according to (12).
5. Recovering the factorization of the MF G by means of formulas (15).

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Ա. Վ. Սարգսյան

Երկրորդ կարգի որոշ դասի մատրից-ֆունկցիաների ֆակտորացման մասին

Աշխատանքում առաջարկվում է երկրորդ կարգի որոշ դասի մատրից-ֆունկցիաների ֆակտորացման եղանակ: Այդ մատրից-ֆունկցիաների ստորին ձախ տարրը գրվում է մատրից-ֆունկցիայի մնացած երեք տարրերի և երկու մերոմորֆ ֆունկցիաների համակցությամբ, որոնք համապատասխանաբար որոշված են միավոր շրջանագծի արտաքին և ներքին տիրույթներում:

А. В. Саргсян.

О факторизации одного класса матриц-функций второго порядка

В работе предлагается метод построения факторизации одного класса матриц-функций второго порядка. Левый нижний элемент этих матриц-функций записывается с помощью комбинаций остальных трех элементов и двух мероморфных функций, которые определяются соответственно внутри и вне единичного круга.