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BUCKLING OF ISOTROPIC PLATES WITH TWO OPPOSITE SIMPLY SUPPORTED EDGES AND THE OTHER TWO EDGES ROTATIONALLY RESTRAINED UNLOADED

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The paper presents buckling loads of isotropic rectangular plates with two simply supported opposite edges and the other two edges elastically supported against rotation. An analytical method that uses the Lévy solution method is employed to determine the buckling loads of the mentioned rectangular plates. The convergence and comparison of the results with those available in the literature indicate the accuracy and the validity of the proposed technique. Effects of the elastic restraint parameters on the mode shapes are illustrated in graphic forms.

***Keywords*:** buckling, elastic restraint, isotropic plates, rectangular plates.

Introduction. Plates of different shapes with different boundary conditions having various applied in-plane force distributions, as well as different buckling factors are considered and documented in [1–4]. Buckling of isotropic plate is discussed in numerous classical monographs. Solution procedures, development of characteristic equations and graphical presentation of buckling curves in terms of dimensionless buckling coefficient and edge restraint coefficient, are well known. Effect of the rotational restraint on the buckling load is thoroughly studied in literature as well. This paper follows the previous papers and presents in standard form parametric information on buckling.

Analysis. Consider the equation governing buckling deflection w (stability equation) for a rectangular isotropic plate, subjected to distributed compressive load P along the x-axis. The rectangular plate simply supported along the edges $x=0$ and $x=a$, and elastically supported against rotation along the other edges is shown in Fig. 1.

$$D\Delta^2 w + P\frac{\partial^2 w}{\partial x^2} = 0, \tag{1}$$

$$w=0, \quad \frac{\partial^2 w}{\partial x^2}=0 \quad \text{at } x=0,a. \tag{2}$$

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The boundary conditions on the rotationally restrained edges at $y=\pm 0,5b$ are

$$\left.\begin{aligned} & w_{yy}+\frac{K_R}{D}w_y=0, \\ & w=0 \end{aligned}\right\} \quad \text{at} \quad y=\pm 0,5b.$$

Here D is a flexural stiffness, defined as $D=\dfrac{Eh^3}{12(1-\nu^2)}$, where E is the Young's modulus, ν is the Poisson's ratio, w is the deflection, Δ is the Laplace operator, and K_R is the restraining moment along the rotationally restrained edge per unit length and per unit rotation [5]. The dimensionless coefficient of rotational restraint R (alternatively identified as ε in the literature) can be defined as $R=\dfrac{K_R b}{D}$, where b is the plate's width. The boundary condition on the restrained edges can then be written as

$$\left.\begin{aligned} & w_{yy}+\frac{R}{b}w_y=0, \\ & w=0 \end{aligned}\right\} \quad \text{at} \quad y=\pm 0,5b. \tag{3}$$

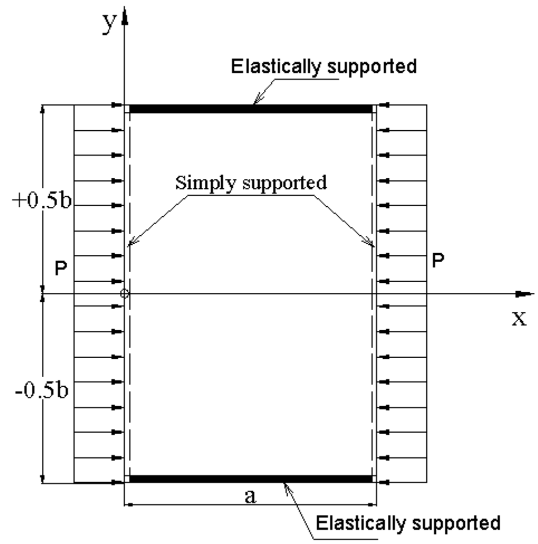

Fig. 1. Uniformly compressed rectangular plate simply supported along the edges x=0 and x=a, and elastically supported against rotation along the other edges (SSEE plate).

The solution to (1) is chosen in a form that satisfies (2)

$$w(x,y)=\sum_{m=1}^{\infty} f_m(y)\sin(\frac{m\pi}{a}x), \quad \mu_m=\frac{m\pi}{a}. \tag{4}$$

The function $f_m(y)$ is the eigenfunction, corresponding to the least eigenvalue (which is to be determined), represents the buckling shape along the y-axis. m represents the number of half-waves in the direction x. Substituting (4) into (1), we find for $f_m(y)$ the following linear ordinary differential equation:

$$\frac{d^4 f_m}{dy^4} - 2\mu_m{}^2 \frac{d^2 f_m}{dy^2} + \left[\mu_m{}^4 - \frac{P}{D} \mu_m{}^2 \right] f_m = 0. \qquad (5)$$

Substituting (4) into (3), we obtain

$$\left. \begin{aligned} &\frac{d^2 f_m}{dy^2} + \frac{R}{b} \cdot \frac{df_m}{dy} = 0, \\ &f = 0 \end{aligned} \right\} \quad \text{at} \quad y = \pm 0,5b. \qquad (6)$$

The form of the solution to (5) depends on the nature of the roots λ of the equation $\lambda^4 - 2\mu_m{}^2 \lambda^2 + \left[\mu_m{}^4 - \frac{P}{D} \mu_m{}^2 \right] = 0$.

Assuming that $\frac{P}{D} > \mu_m{}^2$, the general solution is

$$f_m(y) = C_1 \cosh \lambda_1 y + C_2 \sinh \lambda_1 y + C_3 \cos \lambda_2 y + C_4 \sin \lambda_2 y, \qquad (7)$$

where

$$(\lambda_1)^2 = \sqrt{\mu_m{}^2 \frac{P}{D}} + \mu_m{}^2, \quad (\lambda_2)^2 = \sqrt{\mu_m{}^2 \frac{P}{D}} - \mu_m{}^2. \qquad (8)$$

Since the deflection w at the buckling load must be a symmetric function of y, in the right hand side of (7) there remain only the first and the third terms. Thus,

$$f_m(y) = C_1 \cosh \lambda_1 y + C_3 \cos \lambda_2 y, \qquad (9)$$

and the general mode shape is given by $w(x,y) = \sum_{m=1}^{\infty} (C_1 \cosh \lambda_1 y + C_3 \cos \lambda_2 y) \sin(\mu_m x)$.

The unknown constants C_1 and C_3 are determined from the edge conditions at $y = \pm 0,5b$. Substituting (9) into (6) and considering $w_{xx} = 0$ at $y = \pm 0,5b$, a set of simultaneous equations with regard to C_1 and C_3 is obtained:

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{Bmatrix} C_1 \\ C_3 \end{Bmatrix} = 0.$$

Here $A_{11} = \lambda_1{}^2 \cosh(\lambda_1 b/2) + (R/b)\lambda_1 \sinh(\lambda_1 b/2)$, $A_{12} = -\lambda_2{}^2 \cos(\lambda_2 b/2) - -(R/b)\lambda_2 \sin(\lambda_2 b/2)$, $A_{21} = \cosh(\lambda_1 b/2)$, $A_{22} = \cos(\lambda_2 b/2)$.

The condition $\Delta = 0$ yields the following characteristic buckling equation:

$$\Delta = A_{11}A_{22} - A_{12}A_{21} = 0,$$

$$\begin{aligned} &\lambda_1{}^2 \cos(\lambda_2 b/2)\cosh(\lambda_1 b/2) + (R/b)\lambda_1 \cos(\lambda_2 b/2)\sinh(\lambda_1 b/2) + \\ &+\lambda_2{}^2 \cosh(\lambda_1 b/2)\cos(\lambda_2 b/2) + (R/b)\lambda_2 \cosh(\lambda_1 b/2)\sin(\lambda_2 b/2) = 0. \end{aligned} \qquad (10)$$

Since λ_1 and λ_2 contain P (8), (10) can be solved, using an iterative scheme, for the smallest P, denoted by P_{cr}, once the geometric and material parameters of the plate are known. The critical buckling may be written as $P_{cr} = k\frac{\pi^2 D}{b^2}$, where k is a numerical factor (buckling coefficient), depending on the plate aspect ratio and material properties.

The transcendental equation is solved to determine the buckling coefficient, *k*, as a function of plate properties, the value of the edge restraint coefficient R, the plate aspect ratio a/b and the mode number m. The buckling mode is given by the mode number, for which the smallest k is obtained for a given set of parameters.

Results and Discussion. Matlab program was written to perform the parametric studies reported below. Parametric studies were performed to investigate the influence of material properties, rotational restraint and mode number on the buckling characteristics of this plate. Buckling load $\overline{P}$ was obtained for different coefficients of rotational restraint values as a function of the plate aspect ratio a/b. Buckling curves are shown in Fig. 2. Due to the fact that the rotational restraint is a variable parameter $R = 0,\ 4,\ 10,\ 30,\ 10000$, $R = 0$ corresponds to a simple support, and $R = \infty$ corresponds to a clamped support. Intermediate values of R imply partial rotational restraint. In the numerical studies $R = 10000$ was taken to represent $R = \infty$ [4, 5].

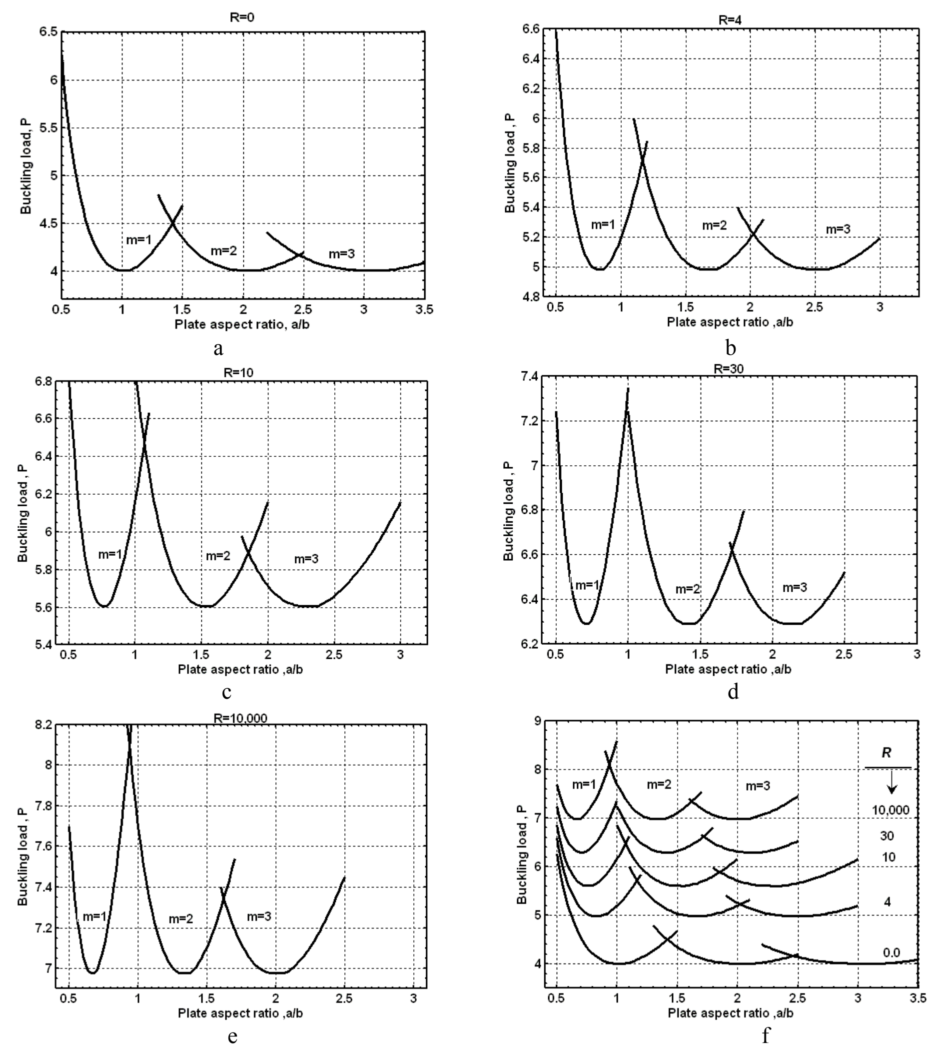

Fig. 2. Non-dimensional buckling loads, $\overline{P} = Pb^2/\pi^2 D$, versus plate aspect ratio a/b:
a) $R = 0$; b) $R = 4$; c) $R = 10$; d) $R = 30$; e) $R = 10{,}000$; f) $0 \le R \le 10{,}000$.

Conclusions. Buckling of an isotropic plate, free supported on its loaded edges, and rotationally restrained on its unloaded edges has been considered in this paper. The correct form of the characteristic transcendental equation for this buckling problem has been provided. Parametric studies have been conducted and buckling curves have been presented.

- $R = 10000 \equiv \infty$. The minimum value of the buckling load $\overline{P} = 6{,}976$ occurs at $a/b = 0{,}665$. There is a mode change at $a/b = 0{,}936$ from $m = 1$ to $m = 2$, the buckling load for this aspect ratio is $\overline{P} = 8{,}095$, and the minimum buckling load $\overline{P} = 6{,}976$ for mode $m = 2$ occurs at $a/b = 1{,}329$. These results corresponds to a clamped support [4].
- $R = 0$. For this case the minimum value of the buckling load $\overline{P} = 4{,}008$ occurs at $a/b = 1$. The result corresponds to exact a simply support [4].
- $0 < R < 10000$. The other buckling curves are between two sets of the mentioned buckling load.

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Ռ. Շարիֆիան

Իզոտրոպ սալերի կայունության կորուստը, երբ սալի երկու կողմերը ազատ հենված են, իսկ մյուս երկուսը ամրակցված են առաձգական հոդակապով

Ներկայացվում են կայունության կորստի կրիտիկական բեռները իզոտրոպ ուղղանկյուն սալերի համար, երբ սալի երկու կողմերը ազատ հենված են, իսկ մյուս երկուսն ամրակցված են առաձգական հոդակապով: Կիրառվում է վերլուծական եղանակ` հիմնված Լևիի աշխատանքի վրա: Կատարված են համեմատություններ գրականությունից հայտնի մասնավոր դեպքերի հետ: Ուսումնասիրված է կրիտիկական բեռի կախվածությունն առաձգական հոդակապի բնութագրիչներից:

Р. Шарифиан.

Устойчивость изотропных пластин в случае, когда две противоположные стороны пластины свободно оперты, а две другие закреплены посредством упругого шарнира.

С помощью аналитического метода Леви определены критические нагрузки потери устойчивости прямоугольных пластин, когда две противоположные стороны пластины свободно оперты, а две другие закреплены посредством упругого шарнира. Изучена зависимость критических нагрузок от параметров упругого закрепления. Для частных случаев установлено соответствие с известными из литературы результатами.