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# VACUUM FLUCTUATIONS IN COSMOLOGICAL MODELS WITH COMPACTIFIED DIMENSIONS

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We study quantum effects of scalar fields in cosmological models of Friedman–Robertson–Walker with a power-low scale factor and spatial topology $R^p \times (S^1)^q$. Recurrent formulae are obtained for positive-frequency Wightman function, vacuum expectation values of the field squared and energy density.

***Keywords*****:** cosmology, vacuum fluctuations, Kaluza–Klein theories.

**1. Introduction.** It is expected that in string theory the most natural topology for the universe is that of a flat compact three-manifold [1]. In inflationary scenario universes with compact spatial dimensions, under certain conditions, should be considered a rule rather than an exception [2]. The models of a compact universe with nontrivial topology may play an important role by providing proper initial conditions for inflation (on the cosmological consequences of the non-trivial topology and observational bounds on the size of compactified dimensions see, for example, [3]). The quantum creation of the universe with toroidal spatial topology is discussed in [4–8] within the framework of various supergravity theories. Vacuum expectation values of the field squared has been considered in the previous work [9].

The compactification of spatial dimensions leads to the modification of the spectrum of vacuum fluctuations and, as a result, to Casimir-type contributions to the vacuum expectation values of physical observables (on the topological Casimir effect and its role in cosmology see [10] and references therein). The effects of the toroidal compactification of spatial dimensions in dS space-time on the properties of quantum vacuum for a scalar field with general curvature coupling parameter are investigated in [11]. The one-loop quantum effects for a fermionic field on background of dS space-time with spatial topology $R^p \times (S^1)^q$ are studied in [12]. In the present paper we investigate the effect of the compactification of one of spatial dimensions in the Friedmann–Robertson–Walker (FRW) cosmological models with power-law scale factor. For a scalar field with general curvature coupling parameter we evaluate the vacuum energy density.

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In this paper we consider the Wightman function for the background FRW space-time with topology $R^p \times (S^1)^q$. We decompose this function in two parts: the first one is the corresponding function for the uncompactified FRW space-time, and the second one is induced by the compactness of the spatial dimensions. We use the Wightman function for the evaluation of the vacuum energy density. As the part corresponding to the uncompactified FRW space-time is well-investigated in literature, we are mainly concerned with the topological part. The asymptotic behavior of the latter is investigated in detail in early and late stages of the cosmological evolution.

**2. Wightman Function in FRW Space-time With Compact Spatial Dimensions.** We consider a quantum scalar field with curvature coupling parameter $\xi$ on background of the $(D+1)$-dimensional FRW space-time. The field equation has the form

$$\left(\nabla_l \nabla^l + m^2 + \xi R\right)\varphi = 0, \tag{1}$$

where $\nabla_l$ is the covariant derivative operator. The values of the curvature coupling parameter $\xi = 0$ and $\xi = \xi_D \equiv (D-1)/4D$ correspond to the most important special cases of minimally and conformally coupled fields. We will write the corresponding line element in the form most appropriate for cosmological applications:

$$ds^2 = dt^2 - a^2(t)\sum_{i=1}^{D}(dz^i)^2, \quad a(t) = \alpha t^c. \tag{2}$$

For the further discussion, in addition to the synchronous time coordinate $t$ it is convenient to introduce the conformal time $\tau$ in accordance with

$$t = \left[\alpha(1-c)\tau\right]^{1/(1-c)}, \quad a(t) = \alpha\left[\alpha(1-c)\tau\right]^{c/(1-c)}. \tag{3}$$

Note that $0 \le \tau < \infty$ for $0 < c < 1$, and $-\infty < \tau \le 0$ for $c > 1$. In terms of this coordinate the line element takes conformally flat form:

$$ds^2 = \Omega^2(\tau)\left[d\tau^2 - \sum_{i=1}^{D}(dz^i)^2\right], \quad \Omega^2(\tau) = \alpha^2\left[\alpha(1-c)\tau\right]^{2c/(1-c)}, \tag{4}$$

and the corresponding Ricci scalar has the form

$$R = \frac{Dc\left[(D+1)c-2\right]}{\left[\alpha(1-c)\tau\right]^{2/(1-c)}}. \tag{5}$$

We will assume that the spatial coordinates $z^l$, $l = D_1+1,\ldots,D$, are compactified to $S^1$: $0 \le z^l \le L_l$, and for the other coordinates we have $-\infty < z^l < +\infty$, $l = 1,\ldots,D_1$. Hence, we consider the spatial topology $R^{D_1} \times (S^1)^{D_2}$. For $D_1 = 0$ as a special case we obtain the toroidally compactified FRW space-time. The results obtained here can be used to describe two types of models. For the first one $D = 4$, and it corresponds to the universe with Kaluza–Klein type single extra dimension. For the second model $D = 3$, and the results given below describe how the properties of the universe are changed by one-loop quantum effects, induced by the compactness of a single spatial dimension.

For a scalar field with periodic boundary condition one has $\varphi(\eta, z_{D_1}, z_{D_2} + \boldsymbol{L}_{D_2}) = \varphi(\eta, z_{D_1}, z_{D_2})$, where $\eta = |\tau|$, $z_{D_1} = (z^1,\ldots,z^{D_1})$, $z_{D_2} = (z^{D_1+1},\ldots,z^D)$,

$\boldsymbol{L}_{D_2}=(L_{D_1+1},\ldots,L_D)$. In this paper we are interested in the effects of non-trivial topology on the vacuum expectation value (VEV) of the energy density. This VEV is obtained from the corresponding Wightman function in the coincidence limit of the arguments.

To evaluate the Wightman function we employ the mode-sum formula

$$G^{+}_{p,q}(x,x')=\langle 0|\varphi(x)\varphi(x')|0\rangle=\sum_{\sigma}\varphi_{\sigma}(x)\varphi^{*}_{\sigma}(x'), \tag{6}$$

where $\{\varphi_{\sigma}(x),\varphi^{*}_{\sigma}(x)\}$ is a complete set of positive and negative frequency solutions to the classical field equation and satisfying the periodicity condition along the compactified dimension. The collective index $\sigma$ specifies these solutions. For the problem under consideration and in the case of a massless field the eigenfunctions have the form [9]

$$\varphi_{\sigma}(x)=C_{\sigma}\eta^{b}H^{(2)}_{\nu}(k\eta)e^{i\boldsymbol{k}_p\cdot\boldsymbol{z}_p+i\boldsymbol{k}_q\cdot\boldsymbol{z}_q}, \tag{7}$$

with the notations

$$\boldsymbol{k}_{D_1}=(k_1,\ldots,k_{D_1}),\quad \boldsymbol{k}_{D_2}=(k_{D_1+1},\ldots,k_D),\quad k=\sqrt{\boldsymbol{k}^2_{D_1}+\boldsymbol{k}^2_{D_2}},$$
$$k_l=2\pi n_l/L_l,\quad n_l=0,\pm1,\pm2,\ldots,\quad l=D_1+1,\ldots,D,$$
$$b=\frac{1}{2}\cdot\frac{cD-1}{c-1}, \tag{8}$$

and the order of the Hankel function $H^{(2)}_{\nu}(x)$ is defined by the relation

$$\nu=\frac{1}{2|1-c|}\left\{(cD-1)^2-4\xi Dc\left[(D+1)c-2\right]\right\}^{1/2}. \tag{9}$$

Note that for a conformally coupled field $\nu=1/2$.

The coefficient $C_{\sigma}$ with $\sigma=(\boldsymbol{k}_{D_1},n_{D_1+1},\ldots,n_D)$ is found from the orthonormalization condition:

$$-i\int\sqrt{|g|}g^{00}\left[\varphi_{\sigma}(x)\partial_{\tau}\varphi^{*}_{\sigma'}(x)-\varphi^{*}_{\sigma'}(x)\partial_{\tau}\varphi_{\sigma}(x)\right]d^{D}x=\delta_{\sigma\sigma'}, \tag{10}$$

where the integration goes over the spatial hypersurface $\tau=const$, and $\delta_{\sigma\sigma'}$ is understood as the Kronecker delta for discrete indices and as the Dirac delta-function for continuous ones. This leads to the result

$$|C_{\sigma}|^2=\frac{\left[\alpha|1-c|\right]^{(D-1)c/(c-1)}e^{-(\nu-\nu^{*})\pi i/2}}{2^{p+2}\pi^{p-1}\alpha^{D-1}V_q},\quad V_q=L_{p+1}\cdots L_D. \tag{11}$$

Substituting the eigenfunctions (7) with the normalization coefficient (11) into the mode-sum formula for the Wightman function, one finds

$$G^{+}_{p,q}(x,x')=\frac{A(\eta\eta')^{b}}{2^{p}\pi^{p+1}V_q}\times$$
$$\times\int e^{i\boldsymbol{k}_p\cdot\Delta\boldsymbol{z}_q}\sum_{\boldsymbol{n}_q=-\infty}^{\infty}e^{i\boldsymbol{k}_q\cdot\Delta\boldsymbol{z}_q}K_{\nu}(k\eta e^{\mathrm{sign}(\tau)\pi i/2})K_{\nu}(k\eta' e^{-\mathrm{sign}(\tau)\pi i/2})\,d\boldsymbol{k}_p, \tag{12}$$

where $\Delta z^{l}=z^{l}-z'^{l}$, $k=\sqrt{\boldsymbol{k}^2_p+\boldsymbol{k}^2_{\boldsymbol{n}_q-1}+k^2_{p+1}}$ and

$$A=\alpha^{1-D}\left[\alpha|1-c|\right]^{(D-1)c/(c-1)}. \tag{13}$$

In (12) we wrote the Hankel function in terms of the MacDonald function $K_\nu(z)$. It can be seen that after the application of the Abel–Plana summation formula [10, 13] to the series over $n_{D_1+1}$, the following recurrence formula is obtained:

$$G^+_{p,q}(x,x') = G^+_{p+1,q-1}(x,x') + \Delta_{p+1} G^+_{p,q}(x,x'), \tag{14}$$

where the first term on the right is the Wightman functions in the FRW space-time with $p+1$ uncompactified and $q-1$ toroidally compactified dimensions, and the second term is induced by the compactness of the $z^{p+1}$-direction and is given by the formula

$$\Delta_{p+1} G^+_{p,q}(x,x') = \frac{A(\eta\eta')^b}{(2\pi)^{p+1} V_{q-1}} \int e^{i\boldsymbol{k}_p\cdot\Delta \boldsymbol{z}_p} \sum_{\boldsymbol{n}_{q-1}=-\infty}^{\infty} e^{i\boldsymbol{k}_{q-1}\cdot\Delta \boldsymbol{z}_{q-1}} d\boldsymbol{k}_p \times$$
$$\times \int_0^\infty \frac{y\cosh\left(\Delta z^{p+1}\sqrt{y^2+\boldsymbol{k}_p^2+\boldsymbol{k}^2_{\boldsymbol{n}_q-1}}\right)}{\sqrt{y^2+\boldsymbol{k}_p^2+\boldsymbol{k}^2_{\boldsymbol{n}_q-1}}\left(e^{L_{p+1}\sqrt{y^2+\boldsymbol{k}_p^2+\boldsymbol{k}^2_{\boldsymbol{n}_q-1}}}-1\right)} \times \tag{15}$$
$$\times \left\{K_\nu(\eta y)\left[I_{-\nu}(\eta' y)+I_\nu(\eta' y)\right]+\left[I_\nu(\eta y)+I_{-\nu}(\eta y)\right]K_\nu(\eta' y)\right\} dy,$$

where $V_{q-1} = L_{p+2}\cdots L_D$ and the notation

$$\boldsymbol{k}^2_{\boldsymbol{n}_q-1} = \sum_{l=p+2}^{D} \left(2\pi n_l/L_l\right)^2 \tag{16}$$

is introduced. Note that in this formula the integration with respect to the angular part of $\boldsymbol{k}_p$ can be done explicitly.

**3. Vacuum Energy Density.** Now we turn to the investigation of the VEV for the vacuum energy density. Using the Wightman function we can evaluate this VEV by making use of the formula [14]

$$\langle 0|T_{00}|0\rangle = \lim_{x'\to x} \partial_0\partial'_0 G^+(x,x') + \left[\left(\xi-\frac{1}{4}\right)g_{00}\nabla_l\nabla^l - \xi\nabla_0\nabla_0 - \xi R_{00}\right]\langle 0|\varphi^2|0\rangle, \tag{17}$$

where $R_{ik}$ is the Ricci tensor for the FRW space-time with the 00-component

$$R_0^0 = \frac{Dc\Omega^{-2}}{(1-c)\tau^2},$$

As in the case of the Wightman function, the renormalized VEV of the energy density is presented as the sum

$$\left\langle T_0^0\right\rangle_{p,q} = \left\langle T_0^0\right\rangle_{p+1,q-1} + \Delta_{p+1}\left\langle T_0^0\right\rangle_{p,q}, \tag{18}$$

where the part due to the compactness of the $z^{p+1}$-direction is given by the expressions

$$\Delta_{p+1}\left\langle T_0^0\right\rangle_{p,q} = \frac{2^{(1-p)/2} A\Omega^{-2}}{\pi^{(p+3)/2} V_{q-1}} \sum_{\boldsymbol{n}_{q-1}=-\infty}^{\infty} \int_0^\infty y^{3-2b} \sum_{n=1}^{\infty} \frac{f_{(p-1)/2}\left(nL_{p+1}\sqrt{y^2+\boldsymbol{k}^2_{\boldsymbol{n}_q-1}}\right)}{\left(nL_{p+1}\right)^{p-1}} F^{(0)}(\eta y)\, dy, \tag{19}$$

with the notation

$$F^{(0)}(z)=\frac{1}{2}\tilde{I}'_{\nu}(z)\tilde{K}'_{\nu}(z)+\frac{D\xi c}{z(1-c)}\left(\tilde{I}(z)\tilde{K}(z)\right)'-\frac{1}{2}\left[1+D\xi c\frac{4-(D+3)c}{z^{2}(1-c)^{2}}\right]\tilde{I}_{\nu}(z)\tilde{K}_{\nu}(z). \quad (20)$$

In (20) we have defined the functions

$$f_{\nu}(x)=x^{\nu}K_{\nu}(x),\quad \tilde{K}_{\nu}(z)=z^{b}K_{\nu}(z),\quad \tilde{I}_{\nu}(z)=z^{b}\left[I_{\nu}(z)+I_{-\nu}(z)\right]. \quad (21)$$

After the recurring application of formula (18), the vacuum energy density in FRW model with spatial topology $R^{p}\times\left(S^{1}\right)^{q}$ is presented in the form

$$\left\langle T_{0}^{0}\right\rangle_{p,q}=\left\langle T_{0}^{0}\right\rangle_{FRW}+\left\langle T_{0}^{0}\right\rangle_{c},$$

where $\left\langle T_{0}^{0}\right\rangle_{FRW}$ is the corresponding quantity for uncompactified FRW space-time and the part

$$\left\langle T_{0}^{0}\right\rangle_{c}=\sum_{l=1}^{q}\Delta_{D-l+1}\left\langle T_{0}^{0}\right\rangle_{D-l,l} \quad (22)$$

is induced by the toroidal compactification of the $q$-dimensional subspace.

For a conformally coupled massless scalar field one has $\nu=1/2$ and $\left[I_{-\nu}(x)+I_{\nu}(x)\right]K_{\nu}(x)=1/x$. For the function $F^{(0)}(z)$ we have:

$$F^{(0)}(z)=z^{2b-3}\left[\frac{c(1-D)}{2(1-c)}-z^{2}\right]. \quad (23)$$

Using the formula

$$\int_{0}^{\infty}f_{(p-1)/2}\left(a\sqrt{z^{2}+b^{2}}\right)dz=\sqrt{\frac{\pi}{2}}\cdot\frac{f_{p/2}(ab)}{a}, \quad (24)$$

for the case of a conformally coupled field we find

$$\Delta_{p+1}\left\langle T_{0}^{0}\right\rangle_{p,q}=-\frac{2\Omega^{-D-1}}{(2\pi)^{p/2+1}V_{q-1}}\sum_{\mathbf{n}_{q-1}=-\infty}^{\infty}\sum_{n=1}^{\infty}\frac{f_{p/2+1}\left(nL_{p+1}k_{\mathbf{n}_{q-1}}\right)}{\left(nL_{p+1}\right)^{p+2}}. \quad (25)$$

Formula (25) could also be obtained from the corresponding result in $(D+1)$-dimensional Minkowski space-time with spatial topology $R^{p}\times\left(S^{1}\right)^{q}$, taking into account that two problems are conformally related: $\Delta_{p+1}\left\langle T_{0}^{0}\right\rangle_{p,q}=\Omega^{-D-1}\Delta_{p+1}\left\langle T_{0}^{0}\right\rangle_{p,q}^{(M)}$. A similar formula takes place for the total topological part.

The general formulas for the topological part in the VEV of the energy density are simplified in the asymptotic regions of the parameters. For small values of the ratio $L_{p+1}/\eta$ we can see that to the leading order $\Delta_{p+1}\left\langle T_{0}^{0}\right\rangle_{p,q}$ coincides with the corresponding result for a conformally coupled massless field given by formula (25). Note that in terms of the synchronous time coordinate we have $L_{p+1}/\eta=\alpha|1-c|L_{p+1}t^{c-1}$ and, hence, $\Delta_{p+1}\left\langle T_{0}^{0}\right\rangle_{p,q}\propto t^{-c(D+1)}$. Hence, the limit under consideration corresponds to the early stages of the cosmological expansion ($t\to 0$) in the case $c>1$ and to the late stages ($t\to+\infty$) in the case $c<1$.

For large values of the ratio $L_{p+1}/\eta$ and in the case of real $\nu$, using the asymptotic formulae for the modified Bessel functions for small values of the argument, to the leading order one has

$$F^{(0)}(z) \approx \frac{2^{2\nu-1} D\Gamma(\nu)}{\Gamma(1-\nu) z^{2\nu}} \left[ D/2 - \nu + 2\xi(2\nu - D - 1) \right]. \quad (26)$$

From formula (19) we find

$$\Delta_{p+1} \left\langle T_0^0 \right\rangle_{p,q} \approx \frac{2^{q+\nu-1} D\left[ D/2 - \nu + 2\xi(2\nu - D - 1) \right]}{(2\pi)^{(p+3)/2} V_q L_{p+1}^{p=2\nu} \alpha^{D+1} \eta^{2\nu-D}} \Gamma(\nu) \sum_{n,\boldsymbol{n}_{q-1}} \frac{f_{(p+1)/2-\nu}\left( nL_{p+1} k_{\boldsymbol{n}_{q-1}} \right)}{n^{(p+1)/2-\nu}}. \quad (27)$$

In terms of the synchronous time coordinate one has $\Delta_{p+1} \left\langle T_0^0 \right\rangle_{p,q} \infty\, t^{(D-2\nu)(1-c)}$. For small values $\eta/L_{p+1}$ and imaginary $\nu$, in a similar way as in the previous case, in terms of the synchronous time coordinate we find

$$\Delta_{p+1} \left\langle T_0^0 \right\rangle_{p,q} \approx \frac{2^{q-1} D|1-c|^{-D} B_0}{(2\pi)^{(p+3)/2} V_q L_{p+1}^{p}} \frac{t^{D(1-c)}}{\alpha^{2D+1}} \sin\left[ 2|\nu| t/\alpha + |\nu| \ln\left( L_{p+1}/\alpha \right) + \varphi_0 \right], \quad (28)$$

where $B_0$ and $\varphi_0$ are defined by the relation

$$B_0 e^{i\varphi_0} = 2^{i|\nu|} \frac{|\nu|(1/2 - 2\xi) + i\left[ D/4 - (D+1)\xi \right]}{\Gamma(1 - i|\nu|)} \sum_{\boldsymbol{n}_{q-1}=-\infty}^{\infty} \sum_{n=\infty}^{\infty} \frac{f_{(p+1)/2-i|\nu|}\left( nL_{p+1} k_{\boldsymbol{n}_{q-1}} \right)}{n^{p+1-2i|\nu|}}. \quad (29)$$

This limit corresponds to the late stages of the cosmological expansion $(t \to +\infty)$ in the case $c > 1$ and to the early stages $(t \to 0)$ in the case $c < 1$.

**4. Conclusion.** Compactified spatial dimensions appear in various physical models, including Kaluza–Klein type theories, supergravity, string theory and cosmology. In this paper we investigate the quantum vacuum effects in FRW space-time induced by non-trivial topology of spatial dimensions. We consider a scalar field with general curvature coupling parameter, satisfying the periodic boundary condition along the compactified dimensions. Among the most important characteristics of the vacuum are the VEV of the energy density. Though the corresponding operator is local, due to the global nature of the vacuum this VEV carry an important information on the global structure of the background space-time.

In order to derive formula for the vacuum energy density, we first construct the Wightman function. Using of the Abel–Plana summation formula, we have extracted from this function the part, corresponding to the Wightman function for the uncompactified FRW space-time. As the topological part is finite in the coincidence limit, by this way the renormalization procedure is reduced to that for the standard FRW case. The latter was already realized in literature [4]. As a result the vacuum energy density is decomposed into FRW and topological parts. For general values of the curvature coupling parameter the corresponding formula is simplified in the asymptotic regions of small and large values of the ratio $L_l/\eta$. In the first case the leading term in the energy density is the same as that for a conformally coupled field, and the topological part behaves like $t^{-c(D+1)}$. This limit corresponds to the early stages of the cosmological expansion in the case $c > 1$ and to the late stages in the case $c<1$. For large values of the ratio $L_l/\eta$ the behavior of the topological part is different for real and pure imaginary values of the parameter ν. In the first case this part behaves like $t^{(D-2\nu)(1-c)}$, whereas in the second case the decay has an oscillatory nature $t^{D(1-c)}\sin(2|\nu|t/\alpha+\varphi_1)$. This limit corresponds to the late stages of the cosmological expansion when $c>1$ and to the early stages when $c<1$.

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REFERENCES

1. **McInnes B.** Nucl. Phys. B 692, 2004, p. 270; ibid 709, 2005, p. 213; 748, 2006, p. 309.
2. **Linde A.** JCAP 0410, 2004, p. 004.
3. **De Oliveira-Costa A., Smoot G.F., Starobinsky A.A.** Astrophys. J., 1996, v. 468, p. 457.
4. **Levin J.** Phys. Rept., 2002, v. 365, p. 251.
5. **Cornish N.J., Spergel D.N., Starkman G.D., Komatsu E.** Phys. Rev. Lett., 2004, v. 92, p. 201–302.
6. **Mostepanenko V.M., Trunov N.N.** The Casimir Effect and Its Applications. Oxford: Clarendon, 1997.
7. **Bordag M., Mohidden U. and Mostepanenko V.M.** Phys. Rept., 2001, v. 353, № 1.
8. **Elizalde E., Odintsov S.D., Romeo A., Bytsenko A.A., Zerbini S.** Zeta Regularization Techniques with Applications. Singapore: World Scientific, 1994.
9. **Saharian A.A.** The Generalized Abel-Plana Formula. Applications to Bessel Functions and Casimir Effect. Report № IC/2000/14; hep-th/0002239.
10. **Birrel N.D. and Davies P.C.W.** Quantum Fields in Curved Space. Cambridge: Cambridge University Press, 1982.
11. **Elizalde E.** J. Phys. A 39, 2006, p. 6299.
12. **Saharian A.A.** Phys. Rev. D 74, 2006, p. 124009.
13. **Zeldovich Y.B., Starobinsky A.A.** Sov. Astron. Lett., 1984, v. 10, p. 135.
14. **Goncharov Yu.P., Bytsenko A.A.** Phys. Lett. B 160, 1985, p. 385; ibid 169, 1986, p. 171.
15. **Goncharov Yu.P., Bytsenko A.A.** Class. Quant. Grav., 1987, v. 4, p. 555.
16. **Saharian A.A., Setare M.R.** Phys. Lett. B 659, 2008, p. 367.
17. **Bellucci S., Saharian A.A.** Phys. Rev. D 77, 2008, p. 124010.
18. **Saharian A.A.** Class. and Quant. Grav., 2008, v. 25, p. 165012.
19. **Bezerra de Mello E.R., Saharian A.A.** JHEP, 2008, v. 12, p. 81.
20. **Mkhitaryan A.L.** Astrophysics, 2009, v. 52, № 4, p. 598.
21. **Saharian A.A.** Phys. Rev. D 69, 2004, p. 85005.

Ա. Լ. Մխիթարյան

### Վակուումային ֆլուկտուացիաները կոմպակտ չափերով կոսմոլոգիական մոդելներում

Ուսումնասիրվում են սկալյար դաշտի քվանտային երևույթները աստիճանային մասշտաբային ֆակտորով Ֆրիդման–Ռոբերթսոն–Ուոկերի կոսմոլոգիական մոդելներում, որոնք ունեն $R^p \times (S^1)^q$ տարածական տոպոլոգիա: Ստացված են Վայթմանի դրական հաճախային ֆունկցիայի անդրադարձ բանաձևեր դաշտի քառակուսու և էներգիայի միջինների համար:

***А. Л. Мхитарян*.**

### Вакуумные флуктуации в космологических моделях с компактными измерениями

Исследованы квантовые эффекты скалярного поля в космологических моделях Фридмана–Робертсона–Уокера со степенным масштабным фактором и с пространственной топологией $R^p \times (S^1)^q$. Получены рекуррентные формулы для положительно-частотной функции Вайтмана и плотности энергии.