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Physics

INVESTIGATION OF THE ELECTRIC AHARONOV–BOHM EFFECT IN A QUANTUM RING

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The bound states of the electron in a quantum ring in the presence of different scalar potentials in two regions are considered. It is shown that the difference of scalar potentials lifts the degeneracy connected with the sign of orbital angular momentum. The same system with a magnetic flux threading the ring is also considered. The possibility of observing the oscillations of energy levels connected with the difference of scalar potentials and the intensity of the magnetic field that are similar to those of the bound states of the electron in a quantum ring threaded only by the magnetic flux is shown.

Keywords: quantum ring, electric Aharonov–Bohm effect, magnetic Aharonov-Bohm effect.

1. Introduction. The original Aharonov–Bohm (AB) effect has a purely quantum mechanical nature showing the important role of the vector and scalar potentials [1]. Although in the original paper by Aharonov and Bohm [1], the electric and magnetic AB effects have been discussed, up to now the majority of both theoretical and experimental works done dealt with the magnetic AB effect. There are only few experimental verifications [2, 3] of the electric AB effect in comparison with the magnetic one. The magnetic bound state AB effect in a quantum ring was considered in the review article [4], and it was shown that the energy levels of charged particle oscillate with magnetic flux $\Phi = \pi r_0^2 B$, if the particle orbits surround an infinitely long solenoid with small radius r_0, where the magnetic field B is concentrated. The magnetic AB effect in a quantum ring was experimentally verified in mesoscopic metal ring [2], carbon nanotubes [5] and doped semiconductor InAs/GaAs nanorings [6].

The aim of the present work is to consider the quantum ring in the presence of different scalar potentials in two regions and describe the motion of the electron inside the ring as a plane wave. This setting does not fully represent those described in the original work by Aharonov and Bohm for general case, because for such a ring the electron interacts with the electric field due to the difference of the potentials. The development of this work will be considering a wave packet, which moves in a quantum ring with time varying scalar potentials in such a way, that the interaction with the electric field is considerably small.

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2. Theory. Consider a quantum ring that is divided into four regions (Fig. 1, a). In two regions (*II* and *IV*) there are two different scalar potentials V_1 and V_2, which create electric fields in two other regions (*I* and *III*) with 2σ arc length (each arc having the corresponding central angle $2\varphi_0$).

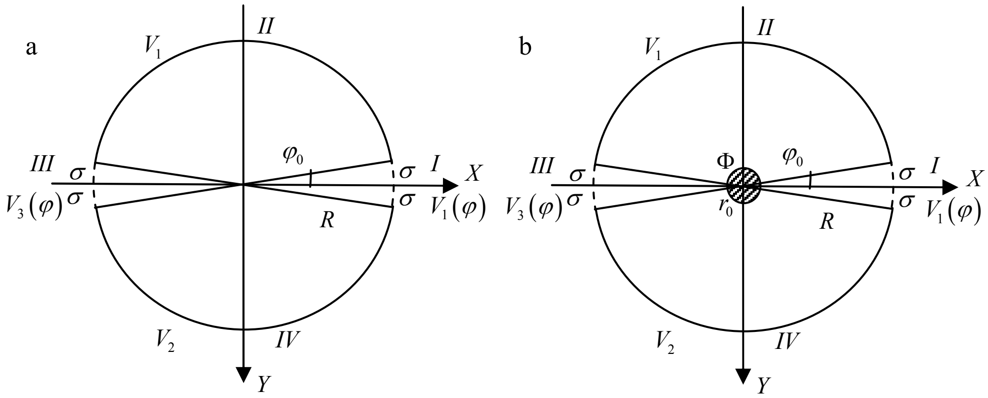

Fig. 1. The electric quantum ring (the electric fields created by the potentials V_1 and V_2 exist in the regions *I* and *III* with arc length 2σ): a) without magnetic flux; b) with the magnetic flux (magnetic field is parallel to Z-direction).

The appropriate electric fields and scalar potentials in the regions *I* and *III* are described by the following expressions:

$$V_1(\varphi)=\frac{(V_1-V_2)}{2\varphi_0}\varphi+\frac{V_1+V_2}{2},\quad -\varphi_0\le\varphi\le\varphi_0\,, \tag{1}$$

$$V_3(\varphi)=\frac{(V_2-V_1)}{2\varphi_0}\varphi+\frac{(\pi+\varphi_0)V_1-(\pi-\varphi_0)V_2}{2\varphi_0},\quad \pi-\varphi_0\le\varphi\le\pi+\varphi_0\,. \tag{2}$$

Using the obtained expressions for scalar potentials for four regions and solving the Schrödinger equation in each region (by introducing the dimensionless units $\rho=R/a_B$, $v_1=V_1/E_R$, $v_2=V_2/E_R$, $\varepsilon=E/E_R$, where a_B and E_R are effective Bohr radius and Rydberg energy respectively), we obtain the following expressions for the wave function in each region:

$$\begin{aligned}\Psi_1(\varphi)=C_1Ai\left(\left(\frac{\rho^2(v_1-v_2)}{2\varphi_0}\right)^{1/3}\left(\varphi+\frac{\varphi_0(2\varepsilon-(v_1+v_2))}{(v_2-v_1)}\right)\right)+\\ +C_2Bi\left(\left(\frac{\rho^2(v_1-v_2)}{2\varphi_0}\right)^{1/3}\left(\varphi+\frac{\varphi_0(2\varepsilon-(v_1+v_2))}{(v_2-v_1)}\right)\right),\end{aligned} \tag{3}$$

$$\Psi_2(\varphi)=C_3\exp\left(i\rho\sqrt{\varepsilon-v_1}\varphi\right)+C_4\exp\left(-i\rho\sqrt{\varepsilon-v_1}\varphi\right), \tag{4}$$

$$\begin{aligned}\Psi_3(\varphi)=C_5Ai\left(\left(\frac{\rho^2(v_1-v_2)}{2\varphi_0}\right)^{1/3}\left(\varphi+\frac{2\varphi_0(\varepsilon-v_1)}{(v_1-v_2)}-\pi+\varphi_0\right)\right)+\\ +C_6Bi\left(\left(\frac{\rho^2(v_1-v_2)}{2\varphi_0}\right)^{1/3}\left(\varphi+\frac{2\varphi_0(\varepsilon-v_1)}{(v_1-v_2)}-\pi+\varphi_0\right)\right),\end{aligned} \tag{5}$$

$$\Psi_4(\varphi)=C_7\exp\left(i\rho\sqrt{\varepsilon-v_2}\varphi\right)+C_8\exp\left(-i\rho\sqrt{\varepsilon-v_2}\varphi\right), \qquad (6)$$

where $Ai(\varphi)$ and $Bi(\varphi)$ are linearly independent Airy functions [7].

In order to find the energy levels of electron in this system, we should impose boundary conditions and the condition of the singlevaluedness on these wave functions. The corresponding conditions are:

$$\begin{aligned} &\Psi_1(\varphi_0)=\Psi_2(\varphi_0), && \Psi_1'(\varphi_0)=\Psi_2'(\varphi_0),\\ &\Psi_2(\pi-\varphi_0)=\Psi_3(\pi-\varphi_0), && \Psi_2'(\pi-\varphi_0)=\Psi_3'(\pi-\varphi_0),\\ &\Psi_3(\pi+\varphi_0)=\Psi_4(\pi+\varphi_0), && \Psi_3'(\pi+\varphi_0)=\Psi_4'(\pi+\varphi_0),\\ &\Psi_4(2\pi-\varphi_0)=\Psi_1(-\varphi_0), && \Psi_4'(2\pi-\varphi_0)=\Psi_1'(-\varphi_0). \end{aligned} \qquad (7)$$

Consider now the same problem in the case when the ring is threaded by a magnetic flux (Fig. 1, b) having a smaller radius than the ring (the magnetic field B is parallel to the Z-direction). We can make gauge transformation in each region and map the problem with magnetic flux to the problem without flux

$$\Psi_{flux}(\varphi)=\Psi(\varphi)\exp\left(-i\frac{\Phi}{\Phi_0}\varphi\right), \qquad (8)$$

where $\Phi_0=hc/e$ is a magnetic flux quantum. Using Eq. (3)–(6) and (8) it is easy to get the expression for wave functions in each region. Using the same conditions as in Eq. (7), we will obtain the energy levels of electron in the ring for this case.

3. Results and Discussion. In Fig. 2 the dependences of the electron energy in the electric quantum ring on the ring radius R (a) and scalar potential V_2 (b) are presented.

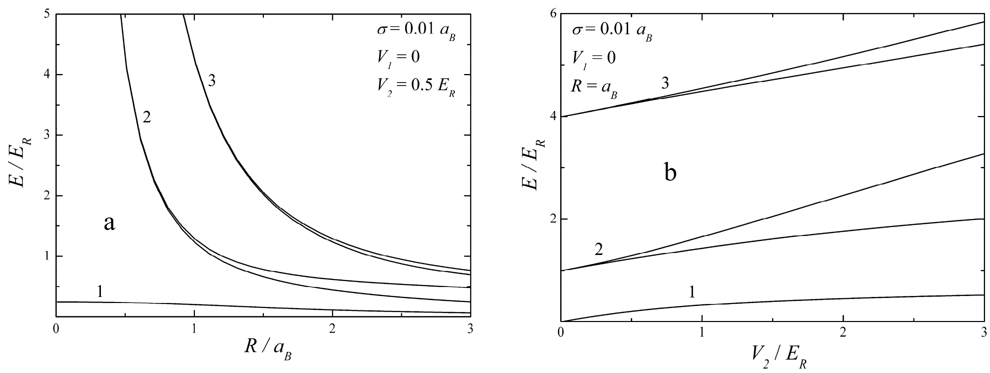

Fig. 2. The electron energy versus the ring radius (a) and the scalar potential V_2 (b).

As is seen in Fig. 2, a, the excited energy levels show the dependence on the ring radius that is similar to the $1/R^2$ dependence for 1D quantum ring and only the ground state (1) shows different behavior and does not go to infinity when $R\to 0$. In order to understand this, let us see that the difference of the scalar potentials in quantum ring is somehow similar to the 1D quantum well, which besides that also has 2π periodicity because of the quantum ring geometry. When $R\to\infty$ the role of the 2π periodicity strongly diminishes and the energy levels tend to the levels of the single quantum well. When $R\to 0$ the quantum ring structure plays more important role, than the existence of the quantum well in it. Because of that, in $R \to 0$ limit the dependence of energy levels on ring radius is described mainly by $1/R^2$ dependence of 1D quantum ring without any scalar potentials. But this is not true for the state with $l = 0$ angular momentum, because this state has zero energy in 1D quantum ring without scalar potentials. Because of that the ground state (which is the level with $l = 0$) in the presence of scalar potentials is mainly influenced by the quantum well energy even for $R \to 0$. And for this reason the ground state tends to a finite value at $R \to 0$. It should be noted that due to the quantum ring structure the energy tends to half the quantum well depth (rather than the full quantum well depth, which is the case for a single quantum well).

As is seen in Fig. 2, b, the difference of scalar potentials lifts the degeneracy of the levels connected with the sign of orbital angular momentum. This is similar to the case of magnetic ring, where the magnetic flux also lifts the degeneracy of $|l|$.

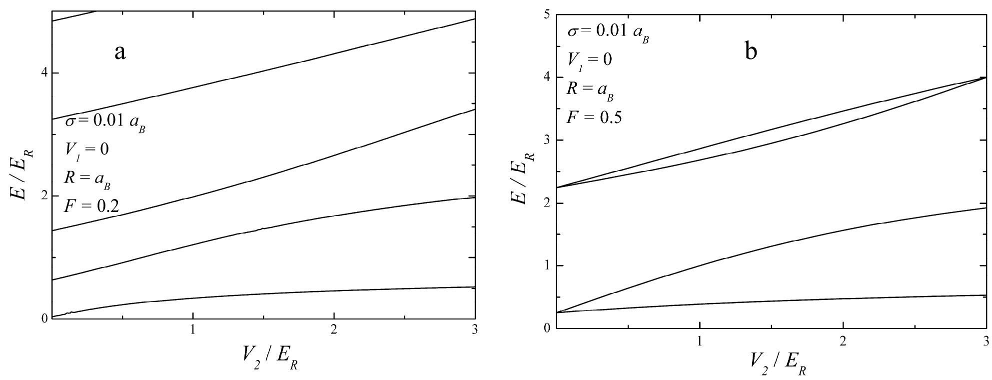

Fig. 3. The dependence of the energy levels on the difference of scalar potentials for different values of the magnetic flux: a) F=0.2; b) F=0.5.

In Fig. 3 the dependences of the energy levels on scalar potential V_2 for different values of the magnetic flux parameter $F = \Phi/\Phi_0$ are presented. As is seen in Fig. 3, the magnetic flux changes the dependence of the energy levels dramatically. When $V_2 = 0$ the energy levels are not degenerate, because of the lifting of degeneracy by the magnetic flux. But in Fig. 3, b the degeneracy for $V_2 = 0$ is again seen and later the difference of scalar potentials again lifts the degeneracy. This is easy to understand from the expression of the energy levels for a magnetic ring case [4, 8]

$$E_{flux} = \frac{\hbar^2}{2mR^2}(l + F)^2 . \tag{9}$$

As is seen from Eq. (9), the energy levels will be degenerate also for cases $l = n$ and $l = -n - 1$ when $F = 0.5$. And this is the degeneracy observed in Fig. 3, b for $V_2 = 0$ case, and further, as in Fig. 2, the difference between scalar potentials again lifts this degeneracy. In Fig. 3, b one may observe also another intersection for $V_2 = 2.953 E_R$ for the third and fourth energy levels. This is somewhat similar to the magnetic ring case, where the oscillations of the energy levels on magnetic flux are observed, although here the oscillations of the degeneracy of the levels are observed and not the oscillations of the dependence on the difference of scalar potentials. This is due to the fact that the electron interacts with the field and the energy of the electron should increase as the difference of scalar potentials is increased. This again gives us the reason to consider the electric quantum ring with time varying scalar potential, where the electron (wave packet) does not interact with the field and the normal oscillations are to be observed. The dependence of energy levels on magnetic flux shown in Fig. 4 also demonstrates the usual oscillations seen in case of magnetic ring.

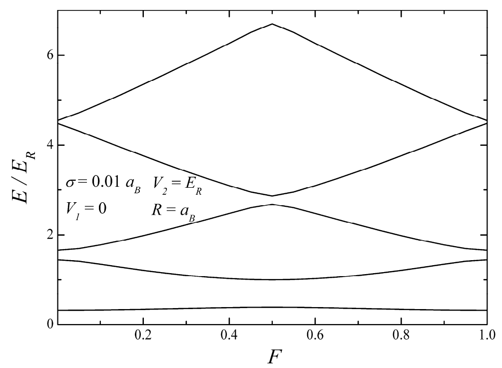

Fig. 4. The dependence of the energy levels on magnetic flux.

4. Conclusion. In this work we have shown that electric AB effect in quantum ring deserves as much interest as the magnetic AB effect. The difference of the scalar potentials lifts the degeneracy of the levels, connected with the sign of the angular momentum in similar manner as the magnetic flux in the magnetic ring case. The investigation of the electric ring with magnetic flux inside has shown that magnetic flux changes the dependences of the energy levels dramatically. However, the settings of the ring used in this work do not fully demonstrate the features of electric AB effect, because the electron interacts with the field. To have the full analog of a magnetic ring for the electric AB effect, it is suggested to consider a wave packet with time varying scalar potentials, so as small interaction with the field be provided.

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REFERENCES

1. **Aharonov Y. and Bohm D.** Phys. Rev., 1959, v. 115, p. 485.
2. **Van Oudenaarden A., Devoret M.H., Nazarov Y.V. and Mooij J.E.** Nature, 1998, v. 391, p. 768.
3. **Schmid H.** In Proc. 8th European Congress on Electron Microscopy. Budapest, 1984.
4. **Peshkin M., Tonomura A.** The Aharonov–Bohm Effect. Lecture Notes in Physics, 1989, v. 340 (eds. Araki H., Ehlers J. et al). Berlin: Springer, p. 10.
5. **Bachtold A., Strunk C., Salvetat J.P., Bonard J.M., Forro L., Nussbaumer T., Schönenberger C.** Nature, 1999, v. 397, p. 673.
6. **Lorke A., Luyken R.J., Govorov A.V., Kotthaus J.P., Garcia J.M. and Petroff P.M.** Phys. Rev. Lett., 2000, v. 84, p. 2223.
7. Handbook of Mathematical Functions, 9th ed. (eds. Abramovitz M. and Stegun I.A.). New York: Dover Publications Inc., 1972.
8. **Hamilton J.** Aharonov–Bohm and other Cyclic Phenomena, 1987, v. 139 (eds. Hohler G., Kuhn J. et al). Berlin: Springer, p. 9.