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PROPAGATION OF A HARMONIC WAVE IN A PLATE WITH SYMMETRIC STRUCTURE

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In this paper a three-layer plate with symmetrical structure to solve the problem of determining the properties of the middle layer, which can propagate the harmonic wave in three-layer plate with phase speed equal to the phase speed of a single-layer plate.

***Keywords*:** sandwich plate, wave, phase speed.

The research of problems about propagation of waves in plates takes the beginning from works Rayleigh [1] and Lemb [2]. The similar problems are investigated in works [3–5]. In work [6] the problem of propagation of a plane harmonic wave in the reinforced layer is considered.

1. Consider the three-dimensional problem of propagation of harmonic waves in elastic sandwich plate with symmetrical structure thickness $2H$. On the lanes, limiting plate, given the condition of vanishing normal stress, a shear stress and a tangential displacement. The middle layer of plate concluded between the planes $z = \pm h$.

For the equation of elastic wave propagation in a plate (the Lame equation)

$$c_{\alpha 2}^2 \Delta \vec{u}_\alpha + (c_{\alpha 1}^2 - c_{\alpha 2}^2)\,\mathrm{grad}\,\mathrm{div}\,\vec{u}_\alpha = \ddot{\vec{u}}_\alpha \tag{1.1}$$

the Lame transformation is introduced

$$\vec{u}_\alpha = \mathrm{grad}\,\varphi_\alpha + \mathrm{rot}\,\vec{\psi}_\alpha \quad (\mathrm{div}\,\vec{\psi}_\alpha = 0)\,. \tag{1.2}$$

Here the following well-known notations are accepted: $c_{\alpha 1} = \left(\dfrac{\lambda_\alpha + 2\mu_\alpha}{\rho_\alpha}\right)^{1/2}$, $c_{\alpha 2} = \left(\dfrac{\mu_\alpha}{\rho_\alpha}\right)^{1/2}$ are respectively the propagation velocities of longitudinal and transverse waves in the material of the layer with number α $(c_{11} = c_{13},\ c_{12} = c_{32})$, α is the number of layer, $\alpha = 1,2,3$, $\vec{u}_\alpha = u_{\alpha 1}\hat{i} + u_{\alpha 2}\hat{j} + u_{\alpha 3}\hat{k}$ is the displacement vector, $\Delta = \partial_1^2 + \partial_2^2 + \partial_3^2$, $\partial_1 = \dfrac{\partial}{\partial x}$, $\partial_2 = \dfrac{\partial}{\partial y}$, $\partial_3 = \dfrac{\partial}{\partial z}$, $\ddot{\vec{u}}_\alpha = \dfrac{\partial^2 \vec{u}_\alpha}{\partial t^2}$, $\varphi_\alpha(x,y,z,t)$, $\vec{\psi}_\alpha(x,y,z,t)$ are the dynamic potentials, λ_α and μ_α are the Lame coefficients.

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At the substitution of (1.2) in equation (1.1) the following wave equations are obtained

$$\Delta\varphi_\alpha - c_{\alpha 1}^{-2}\ddot{\varphi}_\alpha = 0, \quad \Delta\vec{\psi}_\alpha - c_{\alpha 2}^{-2}\ddot{\vec{\psi}}_\alpha = 0. \tag{1.3}$$

The general solution of (1.3) is represented as

$$\begin{aligned} \varphi_\alpha(x,y,z,t) &= \varphi_\alpha^*(z)\cdot \exp i(k_1x + k_2y - ckt), \\ \vec{\psi}_\alpha(x,y,z,t) &= \vec{\psi}_\alpha^*(z)\cdot \exp i(k_1x + k_2y - ckt). \end{aligned} \tag{1.4}$$

Pursuant to (1.4) the equations (1.3) are reduced to ordinary differential equations for unknown functions $\varphi_\alpha^*(z)$ and $\vec{\psi}_\alpha^*(z)$. Solving these equations for dynamic potentials we have

$$\begin{aligned} \varphi_\alpha(x,y,z,t) &= (A_\alpha \operatorname{sh}\nu_{\alpha 1}z + B_\alpha \operatorname{ch}\nu_{\alpha 1}z)\cdot \exp i(k_1x + k_2y - ckt), \\ \vec{\psi}_\alpha(x,y,z,t) &= (\vec{C}_\alpha \operatorname{sh}\nu_{\alpha 2}z + \vec{D}_\alpha \operatorname{ch}\nu_{\alpha 2}z)\cdot \exp i(k_1x + k_2y - ckt), \end{aligned} \tag{1.5}$$

where

$$\nu_{\alpha 1}^2 = k^2(1-\eta_\alpha\theta_\alpha),\ \nu_{\alpha 2}^2 = k^2(1-\eta_\alpha),\ \eta_\alpha = \frac{c^2}{c_{\alpha 2}^2},\ \theta_\alpha = \frac{c_{\alpha 2}^2}{c_{\alpha 1}^2},\ k^2 = k_1^2 + k_2^2,\ \gamma = \frac{c_{22}^2}{c_{12}^2},$$

c is the propagation velocity of the harmonic wave, $A_\alpha, B_\alpha, \vec{C}_\alpha(C_{\alpha 1}, C_{\alpha 2}, C_{\alpha 3})$, $\vec{D}_\alpha(D_{\alpha 1}, D_{\alpha 2}, D_{\alpha 3})$ are unknown constants.

According to the transformation (1.2) and the Hooke law the displacements and stress components are expressed in terms of the dynamic potential by means of the following formulas:

$$\begin{aligned} &u_1 = \partial_1\varphi + \partial_2\psi_3 - \partial_3\psi_2, \quad u_2 = \partial_2\varphi + \partial_3\psi_1 - \partial_1\psi_3, \quad u_3 = \partial_3\varphi + \partial_1\psi_2 - \partial_2\psi_1, \\ &\sigma_{11} = 2\mu(\partial_1^2\varphi + \partial_1\partial_2\psi_3 - \partial_1\partial_3\psi_2) + \lambda\Delta\varphi, \\ &\sigma_{22} = 2\mu(\partial_2^2\varphi + \partial_2\partial_3\psi_1 - \partial_1\partial_2\psi_3) + \lambda\Delta\varphi, \\ &\sigma_{33} = 2\mu(\partial_3^2\varphi + \partial_1\partial_3\psi_2 - \partial_2\partial_3\psi_1) + \lambda\Delta\varphi, \\ &\sigma_{12} = \mu(2\partial_1\partial_2\varphi + \partial_1\partial_3\psi_1 - \partial_2\partial_3\psi_2 + \partial_2^2\psi_3 - \partial_1^2\psi_3), \\ &\sigma_{13} = \mu(2\partial_1\partial_3\varphi + \partial_2\partial_3\psi_3 - \partial_1\partial_2\psi_1 + \partial_1^2\psi_2 - \partial_3^2\psi_2), \\ &\sigma_{23} = \mu(2\partial_2\partial_3\varphi + \partial_1\partial_2\psi_2 - \partial_1\partial_3\psi_3 + \partial_3^2\psi_1 - \partial_2^2\psi_1). \end{aligned} \tag{1.6}$$

For determination of constants $A_\alpha, B_\alpha, \vec{C}_\alpha$ and $\vec{D}_\alpha$ one should use the boundary conditions and the conjugation conditions on the planes $z = \pm h$. Assuming that on the interfacial planes $z = \pm H$ the boundary conditions are specified

$$\sigma_{13} = 0, \quad \sigma_{33} = 0, \quad u_2 = 0. \tag{1.7}$$

The conjugation conditions in case of $z = \pm h$ take the form

$$\vec{u}_1 = \vec{u}_2,\ \ \sigma_{13}^{(1)} = \sigma_{13}^{(2)},\ \ \sigma_{23}^{(1)} = \sigma_{23}^{(2)},\ \ \sigma_{33}^{(1)} = \sigma_{33}^{(2)}. \tag{1.8}$$

Substituting (1.5) in equations (1.6) and using the boundary condition (1.7) and (1.8), we obtain a system of eighteen linear homogeneous equations with constants A_α, B_α, $\vec{C}_\alpha$ and $\vec{D}_\alpha$. The characteristic equation for determination of harmonic wave velocity in a three-layer plate is obtained by equating the determinant of this system to zero. Below we consider two versions that are related to solving the problem of periodic wave propagation in an elastic layer [6].

2. Consider a partial solution for the first layer

$$\begin{aligned}&\varphi_1 = A_1 \mathrm{sh}\,\nu_{11} z \cdot \exp i\,(k_1 x + k_2 y - ckt), \quad \psi_{11} = D_{11} \mathrm{ch}\,\nu_{12} z \cdot \exp i\,(k_1 x + k_2 y - ckt),\\ &\psi_{12} = D_{12} \mathrm{ch}\,\nu_{12} z \cdot \exp i\,(k_1 x + k_2 y - ckt), \quad \psi_{13} = C_{13} \mathrm{sh}\,\nu_{12} z \cdot \exp i\,(k_1 x + k_2 y - ckt)\end{aligned} \tag{2.1}$$

and for the second layer

$$\begin{aligned}&\varphi_2 = B_2 \mathrm{ch}\,\nu_{21} z \cdot \exp i(k_1 x + k_2 y - ckt), \quad \psi_{21} = C_{21} \mathrm{sh}\,\nu_{22} z \cdot \exp i(k_1 x + k_2 y - ckt),\\ &\psi_{22} = C_{22} \mathrm{sh}\,\nu_{22} z \cdot \exp i(k_1 x + k_2 y - ckt), \; \psi_{23} = D_{23} \mathrm{ch}\,\nu_{22} z \cdot \exp i(k_1 x + k_2 y - ckt).\end{aligned} \tag{2.2}$$

Solution (2.1) for the first layer corresponds to the antisymmetric form vibrations, and (2.2) for the second layer corresponds to the symmetrical form vibrations.

Keeping in mind that $\mathrm{div}\,\vec{\psi}_1 = 0$, $\mathrm{div}\,\vec{\psi}_2 = 0$, we obtain after substitution of (2.1) and (2.2) in equations (1.6) and are regard for boundary conditions (1.7) and (1.8)

$$\begin{aligned}&((\lambda_1 + 2\mu_1)\nu_{11}^2 - \lambda_1 k^2)\mathrm{sh}\,\nu_{11} H \cdot A_1 - 2ik_2\mu_1\nu_{12}\mathrm{sh}\,\nu_{12} H \cdot D_{11} + 2ik_1\mu_1\nu_{12}\mathrm{sh}\,\nu_{12} H \cdot D_{12} = 0,\\ &2ik_1\nu_{11}\mathrm{ch}\,\nu_{11} H \cdot A_1 + ik_2\nu_{12}\mathrm{ch}\,\nu_{12} H \cdot C_{13} + k_1 k_2 \mathrm{ch}\,\nu_{12} H \cdot D_{11} - (k_1^2 + \nu_{12}^2)\mathrm{ch}\,\nu_{12} H \cdot D_{12} = 0,\\ &ik_2 \mathrm{sh}\,\nu_{11} H \cdot A_1 - ik_1 \mathrm{sh}\,\nu_{12} H \cdot C_{13} + \nu_{12}\mathrm{sh}\,\nu_{12} H \cdot D_{11} = 0,\\ &\nu_{12} \cdot C_{13} + ik_1 \cdot D_{11} + ik_2 \cdot D_{12} = 0.\end{aligned} \tag{2.3}$$

$$\begin{aligned}&2ik_1\nu_{11}\mathrm{ch}\nu_{11}h\cdot A_1 + ik_2\nu_{12}\mathrm{ch}\nu_{12}h\cdot C_{13} + k_1k_2\mathrm{ch}\nu_{12}h\cdot D_{11} - (k_1^2+\nu_{12}^2)\mathrm{ch}\nu_{12}h\cdot D_{12} =\\ &= \chi(2ik_1\nu_{21}\mathrm{sh}\nu_{21}h\cdot B_2 + ik_2\nu_{22}\mathrm{sh}\nu_{22}h\cdot D_{23} + k_1k_2\mathrm{sh}\nu_{22}h\cdot C_{21} - (k_1^2+\nu_{22}^2)\mathrm{sh}\nu_{22}h\cdot C_{22}),\\ &2ik_2\nu_{11}\mathrm{ch}\nu_{11}h\cdot A_1 - ik_1\nu_{12}\mathrm{ch}\nu_{12}h\cdot C_{13} - k_1k_2\mathrm{ch}\nu_{12}h\cdot D_{12} + (k_2^2+\nu_{12}^2)\mathrm{ch}\nu_{12}h\cdot D_{11} =\\ &= \chi(2ik_2\nu_{21}\mathrm{sh}\nu_{21}h\cdot B_2 - ik_1\nu_{22}\mathrm{sh}\nu_{22}h\cdot D_{23} - k_1k_2\mathrm{sh}\nu_{22}h\cdot C_{22} + (k_2^2+\nu_{22}^2)\mathrm{sh}\nu_{22}h\cdot C_{21}),\\ &((\lambda_1+2\mu_1)\nu_{11}^2 - \lambda_1k^2)\mathrm{sh}\nu_{11}h\cdot A_1 - 2ik_2\mu_1\nu_{12}\mathrm{sh}\nu_{12}h\cdot D_{11} + 2ik_1\mu_1\nu_{12}\mathrm{sh}\nu_{12}h\cdot D_{12} =\\ &= ((\lambda_2+2\mu_2)\nu_{21}^2 - \lambda_2k^2)\mathrm{ch}\nu_{21}h\cdot B_2 - 2ik_2\mu_2\nu_{22}\mathrm{ch}\nu_{22}h\cdot C_{21} + 2ik_1\mu_2\nu_{22}\mathrm{ch}\nu_{22}h\cdot C_{22},\\ &ik_1\mathrm{sh}\nu_{11}h\cdot A_1 + ik_2\mathrm{sh}\nu_{12}h\cdot C_{13} - \nu_{12}\mathrm{sh}\nu_{12}h\cdot D_{12} = ik_1\mathrm{ch}\nu_{21}h\cdot B_2 + ik_2\mathrm{ch}\nu_{22}h\cdot D_{23} - \nu_{22}\mathrm{ch}\nu_{22}h\cdot C_{22},\\ &ik_2\mathrm{sh}\nu_{11}h\cdot A_1 - ik_1\mathrm{sh}\nu_{12}h\cdot C_{13} + \nu_{12}\mathrm{sh}\nu_{12}h\cdot D_{11} = ik_2\mathrm{ch}\nu_{21}h\cdot B_2 - ik_1\mathrm{ch}\nu_{22}h\cdot D_{23} + \nu_{22}\mathrm{ch}\nu_{22}h\cdot C_{21},\\ &\nu_{11}\mathrm{ch}\nu_{11}h\cdot A_1 + ik_1\mathrm{ch}\nu_{12}h\cdot D_{12} - ik_2\mathrm{ch}\nu_{12}h\cdot D_{11} = \nu_{21}\mathrm{sh}\nu_{21}h\cdot B_2 + ik_1\mathrm{sh}\nu_{22}h\cdot C_{22} - ik_2\mathrm{sh}\nu_{22}h\cdot C_{21},\\ &\nu_{12}\cdot C_{13} + ik_1\cdot D_{11} + ik_2\cdot D_{12} = 0,\\ &\nu_{22}\cdot D_{23} + ik_1\cdot C_{21} + ik_2\cdot C_{22} = 0,\end{aligned} \tag{2.4}$$

where $\chi = \mu_2 / \mu_1$.

For existence of nonzero solutions of (2.3) and (2.4) it is required that their determinants be zero. From this condition we obtain the following dispersion equation:

$$\frac{\mathrm{th}\,\nu_{11} H}{\mathrm{th}\,\nu_{12} H} = \frac{4\sqrt{(1-\eta_1\theta_1)(1-\eta_1)}}{(2-\eta_1)^2 - \xi^2\eta_1(1-\eta_1)}, \tag{2.5}$$

$$\mathrm{th}\,\nu_{12} h \cdot \mathrm{th}\,\nu_{22} h = \frac{\nu_{12}}{\nu_{22}\chi}, \tag{2.6}$$

$$\begin{aligned}&\sqrt{1-\gamma\eta_2}\sqrt{1-\theta_2\eta_2}\,\gamma\eta_2^2\chi a_{11}a_{12}a_{21}a_{22} - \sqrt{1-\theta_2\eta_2}\sqrt{1-\eta_2}\,(2-2\chi-\gamma\eta_2)^2 a_{11}a_{21} +\\ &+((2-\eta_2)\chi + \gamma\eta_2 - 2)^2 a_{11}a_{22} + 4\sqrt{1-\gamma\theta_1\eta_2}\sqrt{1-\theta_2\eta_2}\sqrt{1-\gamma\eta_2}\sqrt{1-\eta_2}\,(-1+\chi)^2 a_{12}a_{21} -\\ &-\sqrt{1-\gamma\theta_1\eta_2}\sqrt{1-\eta_2}\,\gamma\eta_2^2\chi - \sqrt{1-\gamma\theta_1\eta_2}\sqrt{1-\gamma\eta_2}\,(\chi(2-\eta_2)-2)^2 a_{12}a_{22} = 0,\end{aligned} \tag{2.7}$$

where $\xi = k_2 / k_1$, $a_{ij} = \mathrm{th}\,\nu_{ij} h$, $i, j = 1,2$.

From equation (2.5) the wave propagation velocity c is determined as a function of wave numbers k_1, k_2, the plate thickness $2H$ and the dimensionless parameter θ_1. The dispersion relations (2.6) and (2.7) determine the conditions imposed on the middle layer of plate to make possible the propagation of a harmonic wave with phase velocity c. If the ratio H/h, θ_1, θ_2 and γ, as well as the value of the phase velocity c (η_1) of equation (2.5) [7] are known, then from (2.6) and (2.7) χ is found depending on kH. Note that (2.7) is a quadratic equation with respect to χ. Fig. 1 (from (2.6)) and 2 (from (2.7)) show the dependence χ on kH $(0 \le kH \le 9)$ for $H/h = 2$, $\theta_1 = \frac{1}{3}$, $\theta_2 = \frac{1}{3}$ and $\gamma = 4$.

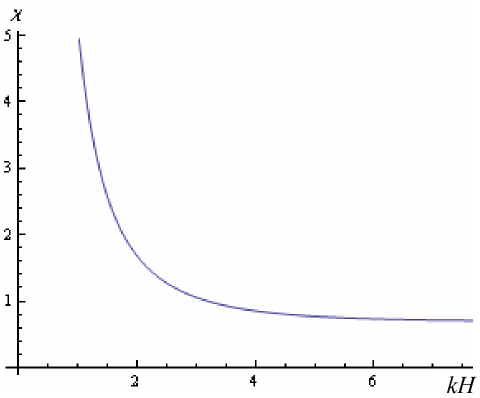

Fig. 1.

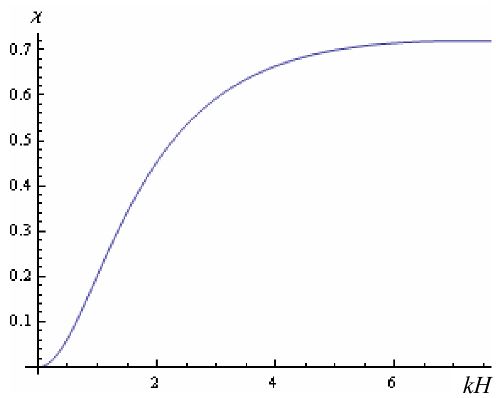

Fig. 2.

3. Now consider the following particular solution for the first layer

$$\begin{aligned} &\varphi_1 = B_1 \operatorname{ch} \nu_{11} z \cdot \exp i(k_1 x + k_2 y - ckt), \quad \psi_{11} = C_{11} \operatorname{sh} \nu_{12} z \cdot \exp i(k_1 x + k_2 y - ckt), \\ &\psi_{12} = C_{12} \operatorname{sh} \nu_{12} z \cdot \exp i(k_1 x + k_2 y - ckt), \; \psi_{13} = D_{13} \operatorname{ch} \nu_{12} z \cdot \exp i(k_1 x + k_2 y - ckt) \end{aligned} \tag{3.1}$$

and for the second layer

$$\begin{aligned} &\varphi_2 = A_2 \operatorname{sh} \nu_{21} z \cdot \exp i(k_1 x + k_2 y - ckt), \quad \psi_{21} = D_{21} \operatorname{ch} \nu_{22} z \cdot \exp i(k_1 x + k_2 y - ckt), \\ &\psi_{22} = D_{22} \operatorname{ch} \nu_{22} z \cdot \exp i(k_1 x + k_2 y - ckt), \; \psi_{23} = C_{23} \operatorname{sh} \nu_{22} z \cdot \exp i(k_1 x + k_2 y - ckt). \end{aligned} \tag{3.2}$$

Solution (3.1) for the first layer corresponds to the symmetrical vibration mode, whereas (3.2) for the second layer corresponds to the antisymmetric vibration mode.

Substituting (3.1) and (3.2) in equations (1.6) and using the boundary conditions (1.7) and (1.8) and having in mind that $\operatorname{div} \vec{\psi}_1 = 0$, $\operatorname{div} \vec{\psi}_2 = 0$, we get:

$$\begin{aligned} &((\lambda_1 + 2\mu_1)\nu_{11}^2 - \lambda_1 k^2) \operatorname{ch} \nu_{11} H \cdot B_1 - 2ik_2 \mu_1 \nu_{12} \operatorname{ch} \nu_{12} H \cdot C_{11} + 2ik_1 \mu_1 \nu_{12} \operatorname{ch} \nu_{12} H \cdot C_{12} = 0, \\ &2ik_1 \nu_{11} \operatorname{sh} \nu_{11} H \cdot B_1 + ik_2 \nu_{12} \operatorname{sh} \nu_{12} H \cdot D_{13} + k_1 k_2 \operatorname{sh} \nu_{12} H \cdot C_{11} - (k_1^2 + \nu_{12}^2) \operatorname{sh} \nu_{12} H \cdot C_{12} = 0, \\ &ik_2 \operatorname{ch} \nu_{11} H \cdot B_1 - ik_1 \operatorname{ch} \nu_{12} H \cdot D_{13} + \nu_{12} \operatorname{ch} \nu_{12} H \cdot C_{11} = 0, \\ &\nu_{12} \cdot D_{13} + ik_1 \cdot C_{11} + ik_2 \cdot C_{12} = 0, \end{aligned} \tag{3.3}$$

$$
\begin{aligned}
&2ik_1\nu_{11}\mathrm{sh}\nu_{11}h\cdot B_1+ik_2\nu_{12}\mathrm{sh}\nu_{12}h\cdot D_{13}+k_1k_2\mathrm{sh}\nu_{12}h\cdot C_{11}-(k_1^2+\nu_{12}^2)\mathrm{sh}\nu_{12}h\cdot C_{12}=\\
&=\chi(2ik_1\nu_{21}\mathrm{ch}\nu_{21}h\cdot A_2+ik_2\nu_{22}\mathrm{ch}\nu_{22}h\cdot C_{23}+k_1k_2\mathrm{ch}\nu_{22}h\cdot D_{21}-(k_1^2+\nu_{22}^2)\mathrm{ch}\nu_{22}h\cdot D_{22}),\\
&2ik_2\nu_{11}\mathrm{sh}\nu_{11}h\cdot B_1-ik_1\nu_{12}\mathrm{sh}\nu_{12}h\cdot D_{13}-k_1k_2\mathrm{sh}\nu_{12}h\cdot C_{12}+(k_2^2+\nu_{12}^2)\mathrm{sh}\nu_{12}h\cdot C_{11}=\\
&=\chi(2ik_2\nu_{21}\mathrm{ch}\nu_{21}h\cdot A_2-ik_1\nu_{22}\mathrm{ch}\nu_{22}h\cdot C_{23}-k_1k_2\mathrm{ch}\nu_{22}h\cdot D_{22}+(k_2^2+\nu_{22}^2)\mathrm{ch}\nu_{22}h\cdot D_{21}),\\
&((\lambda_1+2\mu_1)\nu_{11}^2-\lambda_1k^2)\mathrm{ch}\nu_{11}h\cdot B_1-2ik_2\mu_1\nu_{12}\mathrm{ch}\nu_{12}h\cdot C_{11}+2ik_1\mu_1\nu_{12}\mathrm{ch}\nu_{12}h\cdot C_{12}=\\
&=((\lambda_2+2\mu_2)\nu_{21}^2-\lambda_2k^2)\mathrm{sh}\nu_{21}h\cdot A_2-2ik_2\mu_2\nu_{22}\mathrm{sh}\nu_{22}h\cdot D_{21}+2ik_1\mu_2\nu_{22}\mathrm{sh}\nu_{22}h\cdot D_{22},\\
&ik_1\mathrm{ch}\nu_{11}h\cdot B_1+ik_2\mathrm{ch}\nu_{12}h\cdot D_{13}-\nu_{12}\mathrm{ch}\nu_{12}h\cdot C_{12}=ik_1\mathrm{sh}\nu_{21}h\cdot A_2+ik_2\mathrm{sh}\nu_{22}h\cdot C_{23}-\nu_{22}\mathrm{sh}\nu_{22}h\cdot D_{22},\\
&ik_2\mathrm{ch}\nu_{11}h\cdot B_1-ik_1\mathrm{ch}\nu_{12}h\cdot D_{13}+\nu_{12}\mathrm{ch}\nu_{12}h\cdot C_{11}=ik_2\mathrm{sh}\nu_{21}h\cdot A_2-ik_1\mathrm{sh}\nu_{22}h\cdot C_{23}+\nu_{22}\mathrm{sh}\nu_{22}h\cdot D_{21},\\
&\nu_{11}\mathrm{sh}\nu_{11}h\cdot B_1+ik_1\mathrm{sh}\nu_{12}h\cdot C_{12}-ik_2\mathrm{sh}\nu_{12}h\cdot C_{11}=\nu_{21}\mathrm{ch}\nu_{21}h\cdot A_2+ik_1\mathrm{ch}\nu_{22}h\cdot D_{22}-ik_2\mathrm{ch}\nu_{22}h\cdot D_{21},\\
&\nu_{12}\cdot D_{13}+ik_1\cdot C_{11}+ik_2\cdot C_{12}=0,\ \nu_{22}\cdot C_{23}+ik_1\cdot D_{21}+ik_2\cdot D_{22}=0.
\end{aligned}
\tag{3.4}
$$

For existence of nonzero solutions of (3.3) and (3.4) it is required that their determinants were equal to zero, whence we obtain the following dispersion equation:

$$
\frac{\mathrm{th}\,\nu_{11}H}{\mathrm{th}\,\nu_{12}H}=\frac{(2-\eta_1)^2-\xi^2\eta_1(1-\eta_1)}{4\sqrt{(1-\eta_1\theta_1)(1-\eta_1)}},
\tag{3.5}
$$

$$
\mathrm{th}\,\nu_{12}h\cdot\mathrm{th}\,\nu_{22}h=\frac{\nu_{22}\chi}{\nu_{12}},
\tag{3.6}
$$

$$
\begin{aligned}
&\sqrt{1-\gamma\theta_1\eta_2}\sqrt{1-\eta_2}\,\gamma\eta_2^2\chi a_{11}a_{12}a_{21}a_{22}-\sqrt{1-\gamma\theta_1\eta_2}\sqrt{1-\gamma\eta_2}\,((2-\eta_2)\chi-2)^2a_{11}a_{21}+\\
&+((2-\eta_2)\chi-\gamma\eta_2+2)^2a_{12}a_{21}+4\sqrt{1-\gamma\theta_1\eta_2}\sqrt{1-\theta_2\eta_2}\sqrt{1-\gamma\eta_2}\sqrt{1-\eta_2}\,(-1+\chi)^2a_{11}a_{22}+\\
&+\sqrt{1-\gamma\eta_2}\sqrt{1-\theta_2\eta_2}\,\gamma\eta_2^2\chi-\sqrt{1-\theta_2\eta_2}\sqrt{1-\eta_2}\,(2(1-\chi)-\gamma\eta_2)^2a_{12}a_{22}=0.
\end{aligned}
\tag{3.7}
$$

As was mentioned above, having the values of phase velocity $c\left(\eta_1\right)$ determined from equation (3.5) and of ratio $H/h,\theta_1,\theta_2$, and γ from equations (3.6) and (3.7), one can find χ, i.e. the Lame coefficient of the intermediate layer of the plate, in which the propagation of harmonic wave with phase velocity, equal to that with $c\ (\eta_1)$ of a single-layer plate was possible.

Note, that when the three-layer plate passes into a single layer, the dispersion relations (2.6), (2.7) and (3.6), (3.7) become identities, and in case of $\xi=0$ these relations coincide with the relevant relations given in [6].

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