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# SURFACE WAVES IN PIEZOACTIVE ELASTIC SYTEM OF A LAYER ON A SEMI-SPACE

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A structure consisting of a layer and a semi-space, made of elastic piezoelectric materials, is considered. Unlike the known approaches to the problem, it is assumed that the layer and the semi-space can freely slide relative to each other. The problem of Gulyayev–Bleustein type surface wave propagation is investigated for four different variants of boundary conditions at the external surface of the layer. It is established, that in one case there exist two Gulyayev–Bleustein type waves, in two other cases there exists one such wave for each case, and in the last case there is no surface wave of mentioned type.

***Keywords*****:** surface wave, boundary conditions, piezoelectric materials.

In numerous known publications the propagation of surface waves at the boundary between an elastic layer and semi-space is considered subject to the condition of full (rigid) contact at the interface. A review of these publications is provided in [1]. Of recent publications in this field it is worthwhile to mention [2, 3]. In the present paper the problems of surface wave propagation are considered in case when conditions of sliding contact are established at the boundary between the layer and the semi-space.

**1.** Let the semi-space in the Cartesian coordinate system be defined by the domain $-\infty<x<\infty,\ 0\le y<\infty,\ -\infty<z<\infty,$ and the layer by $-\infty<x<\infty,\ -h\le y\le 0,$ $-\infty<z<\infty$ domain. The layer and the semi-space are made of the same piezoelastic material of 6mm class. Whence, all quantities relating to the semi-space are denoted by index $k=1,$ while all those relating to the layer are denoted by index $k=2.$ Equations of propagation of purely shear electro-elastic waves are well known [4, 5]:

$$a^2\Delta w_k=\partial^2 w_k/\partial t^2,\ \Delta\psi_k=0,\ \ k=1,2, \tag{1.1}$$

where

$$a^2=\tilde{C}_{44}/\rho,\ \tilde{C}_{44}=C_{44}(1+\chi),\ \ \chi=e_{15}^2/\varepsilon C_{44},\ \psi_k=\varphi_k-e_{15}w_k/\varepsilon, \tag{1.2}$$

here $\tilde{C}_{44}$ is the shear modulus, $\rho$ is the mass density of material, $\varepsilon$ is the dielectric permittivity, $e_{15}$ is the piezoelectric modulus, $\chi$ is the electro-mechanical coupling

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coefficient, $\varphi_k$ are electric potentials in the semi-space and in the layer respectively. At the boundary between the layer and the semi-space the conditions of sliding contact are imposed: $\sigma_{23}^{(1)}=\sigma_{23}^{(2)}=0$; $\varphi_1=\varphi_2$; $D_2^{(1)}=D_2^{(2)}$, when $y=0$. We will write this equations, taking into account the functional equations from [5, 6], as

$$\tilde{C}_{44}\frac{\partial w_k}{\partial y}+e_{15}\frac{\partial \psi_k}{\partial y}=0,\quad \varphi_1=\varphi_2,\quad \frac{\partial \psi_1}{\partial y}=\frac{\partial \psi_2}{\partial y},\ \text{when}\ y=0. \tag{1.3}$$

The following four types of boundary conditions are imposed at the external boundary of the layer:

$$\begin{array}{ll} \text{a)}\ \sigma_{23}^{(2)}=0,\ \varphi_2=0; & \text{b)}\ \sigma_{23}^{(2)}=0,\ D_2^{(2)}=0; \\ \text{c)}\ w_2=0,\ \varphi_2=0; & \text{d)}\ w_2=0,\ D_2^{(2)}=0. \end{array} \tag{1.4}$$

The sought for solution of Eqs. (1.1) shall satisfy the interface boundary conditions (1.3), any of the versions of boundary conditions (1.4) and at the same time the following attenuation conditions:

$$\lim_{y\to\infty} w_1=0,\ \lim_{y\to\infty}\psi_1=0. \tag{1.5}$$

The existence of non-trivial solutions to any of mentioned problems would mean the existence of the Gulyayev–Bleustein type surface (or localized) wave [8]. Solutions of Eqs. (1.1) are sought in the form:

$$w_k=f_k(y)\exp i(\omega t-px),\quad \psi_k=g_k(y)\exp i(\omega t-px). \tag{1.6}$$

Substituting (1.6) into (1.1) a set of ordinary differential equations with respect to functions $f_k$, $g_k$ is obtained. General solutions of these equations satisfying the attenuation conditions (1.5) are

$$\begin{array}{l} f_1=A_1e^{-p\alpha y},\ \ f_2=A_2e^{p\alpha y}+B_2e^{-p\alpha y}, \\ g_1=C_1e^{-py},\ \ g_2=C_2e^{py}+D_2e^{-py}, \end{array} \tag{1.7}$$

where $A_1, A_2, B_2, C_1, C_2, D_2$ are arbitrary constants and

$$\alpha=(1-\eta)^{1/2},\ \ \eta=\omega^2(pa)^{-2}. \tag{1.8}$$

Thus, the problem is reduced to determination of parameter $\eta$ characterizing the square of phase velocity subject to the attenuation condition

$$0<\eta<1. \tag{1.9}$$

According to (1.2), (1.6) and (1.7), the potential functions are determined by the following expressions:

$$\varphi_k=(g_k+e_{15}\varepsilon^{-1}f_k)\exp i(\omega t-px). \tag{1.10}$$

Substituting (1.6) and (1.10) into boundary conditions at the interface between the layer and the semi-space, with due regard for (1.7) gives:

$$\begin{array}{l} \alpha\tilde{C}_{44}A_1+e_{15}C_1=0,\ \ \alpha\tilde{C}_{44}(A_2-B_2)+e_{15}(C_2-D_2)=0, \\ C_1+e_{15}\varepsilon^{-1}A_1=C_2+D_2+e_{15}\varepsilon^{-1}(A_2+B_2), \\ C_1=D_2-C_2. \end{array} \tag{1.11}$$

The obtained set (1.11) is a system of four algebraic linear equations with respect to six unknown constants that need to be completed by adding to it the conditions at the external boundary $y=-h$ of layer. However, it is convenient to express four unknown constants in terms of $C_2$ and $D_2$ as follows:

$$A_1 = e_{15}(\tilde{C}_{44}\alpha)^{-1}(C_2 - D_2),\ \ C_1 = D_2 - C_2,$$
$$A_2 = -\varepsilon e_{15}^{-1} C_2,\ \ B_2 = -\varepsilon e_{15}^{-1}(1-\alpha^{-1}\chi_0)C_2 - e_{15}(\tilde{C}_{44}\alpha)^{-1} D_2. \tag{1.12}$$

In (1.12) the quantity $\chi_0$ is determined by electromechanical coupling coefficient: $\chi_0 = e_{15}^2(\varepsilon\tilde{C}_{44})^{-1} = \chi(1+\chi)^{-1}$. The use of convenience expressions (1.12) will significantly simplify the solution of the problem.

**2.** Now let consider the first case, when the external boundary of the layer is free of mechanical stresses and is electrically earthed; so, the boundary conditions are:

$$\tilde{C}_{44}(\partial w_2/\partial y) + e_{15}(\partial\psi_2/\partial y) = 0,\ \ \varphi_2 = 0,\ \text{ when }\ y = -h. \tag{2.1}$$

Substituting $w_2, \psi_2$ from (1.6) and $\varphi_2$ from (1.10) with due regard for (1.7) into boundary conditions (2.1), we get equations with respect to the four unknown constants $A_2, B_2,\ C_2, D_2$:

$$\alpha\tilde{C}_{44}(A_2 e^{-\alpha ph} - B_2 e^{\alpha ph}) + e_{15}(C_2 e^{-ph} - D_2 e^{ph}) = 0,$$
$$e_{15}(A_2 e^{-\alpha ph} + B_2 e^{\alpha ph}) + \varepsilon(C_2 e^{-ph} + D_2 e^{ph}) = 0. \tag{2.2}$$

Using (1.12), the system (2.2) yields to two homogeneous equations with respect to unknown constants $C_2, D_2$. Based on the condition that the determinant of this system is zero, we obtain the following equation for phase velocity of the wave:

$$\left[(\alpha-\chi_0)^2 - \chi_0(\alpha+\chi_0)e^{-2ph} - \alpha(\alpha+\chi_0)e^{-2ph\alpha_0}\right]e^{ph(1+\alpha)} + 4\chi_0\alpha = 0. \tag{2.3}$$

The above mentioned problem was first considered in [6], where it was established that Eq. (2.3) has two roots satisfying the attenuation condition (1.9). An equivalent result was obtained in [7] in case of diffraction of electroelastic shear wave.

Now consider the case when the conditions (1.4, a) are fulfilled at the external boundary of the layer. According to [8], these conditions are reduced to

$$\partial w_2/\partial y = 0,\ \ \partial\psi_2/\partial y = 0,\ \text{ when }\ y = -h. \tag{2.4}$$

Applying the Eqs. (2.4) to the generic solution obtained in previous sections, we derive the following dispersion equation:

$$L(\eta) \equiv \alpha e^{ph}\text{sh}(\alpha ph) - \chi_0 e^{\alpha ph}\text{sh}(ph) = 0. \tag{2.5}$$

For the Eq. (2.5) following inequalities hold:

$$L(0) = (1-\chi_0)e^{ph}\text{sh}(ph) > 0,$$
$$L(1) = -\chi_0\text{sh}(ph) < 0, \tag{2.6}$$

this means that the Eq. (2.5) has at least one real root satisfying the attenuation condition (1.9): i.e. there exists a Gulyayev–Bleustein type wave (subject to condition $ph \neq 0$). It can be shown that the root of Eq. (2.5) in the interval $0 < \eta < 1$ is unique.

For large values of $ph$ (short wave approximation), Eq. (2.5) has a solution:

$$\alpha = \chi_0 \Rightarrow \eta = 1 - \chi^2/(1+\chi)^2, \tag{2.7}$$

that coincides with the velocity of the Gulyayev–Bleustein wave.

For small values of $ph$ (long wave approximation) we obtain:

$$\eta = 1 - \chi/(1+\chi). \tag{2.8}$$

From comparison of (2.7) and (2.8) the velocity of long waves is larger, and, thus, they attenuate faster with the distance from the surface of semi-space.

In the case of fixed and electrically earthed surface of $y=-h$ layer (1.4, c), the following dispersion equation is obtained:

$$M(\eta) \equiv \chi_0 e^{\alpha ph} \mathrm{ch}(ph) - \alpha e^{ph} \mathrm{ch}(\alpha ph) = 0. \tag{2.9}$$

The function $M(\eta)$ satisfies the following inequalities:

$$M(0) = (\chi_0 - 1) e^{ph} \mathrm{ch}(ph) < 0, \qquad M(1) = \chi_0 \mathrm{ch}(ph) > 0, \tag{2.10}$$

i.e. in this case the dispersion equation has an unique root satisfying the condition $0<\eta<1$, also. However, contrary to the previous case, both in the short wave ($ph >> 1$), and the long wave approximation ($ph << 1$), this equation has the root (2.7) that corresponds to the Gulyayev–Bleustein wave.

The case of boundary conditions (1.4, d), is reduced to the form, $w_2 = 0,\ \partial \psi_2 / \partial y = 0,$ when $y=-h.$ The dispersion equation, corresponding to these conditions is $\chi_0 e^{\alpha ph} \mathrm{sh}(ph) + \alpha e^{ph} \mathrm{ch}(ph) = 0.$ It is obvious that this equation has no roots in the interval $0<\eta<1.$

It is concluded, thus, that for boundary conditions (1.4, a) there exist two Gulyayev–Bleustein type surface waves, for boundary conditions (1.4, b) and (1.4, c) there exists by one surface wave in each case, and no surface wave does exist in case of version (1.4, d).

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