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PROBLEM OF MAGNETOELASTIC VIBRATIONS OF PLATE-STRIP IN THE PRESENCE OF EXTERNAL LONGITUDINAL MAGNETIC FIELD

M. V. BELUBEKYAN *, A. A. PAPYAN **

Institute of Mechanics of NAS Republic of Armenia

Bending vibrations of electroconductive plate-strip in the longitudinal magnetic field are being investigated. The problem is solved on the basis of hypothesis of magnetoelasticity of thin bodies by the use of the model of the perfect conductor for boundary condition on the surface faces of plate-strip. The numerical results of the frequency vibrations and damping coefficients are brought, based on the intensity of magnetic field.

***Keywords*:** magnetoelasticity, conductive plate, vibration, frequency.

1. Introduction and Problem Statement. The hypothesis of magneto-elasticity of thin bodies, with the assumption of Kirchhoff's theory, constant along thickness of tangential component of induced electric field and normal component of induced magnetic field is also proposed [1]. On the basis of hypothesis three-dimensional interaction equations of elasticity and electrodynamics in the area occupied by plate are brought to two-dimensional [1–3]. However, in any case, the problem turns to stay three-dimensional, as the solution of electrodynamics around surroundings of plate is required. Only in special case, when the internal magnetic field is perpendicular in the middle of plane of the plate, there is no need to join with electrodynamic filed out of the plate. The results of different approaches to the final diverting of three-dimensional problem of magnetoelastic vibrations of the plate to two-dimensional in the longitudinal magnetic field were proposed [1, 4, 5]. In the present article, on the basis of the used model of the perfect conductor for boundary conditions on surface faces of plate, new way is proposed of bringing to two-dimensional problems.

Elastic isotropic plate-strip with the constant thickness $2h$ and with the finite electroconductive σ is in the external magnetic field $\vec{H}_0 = \vec{H}_0(H_{01}, 0, 0)$. Plate-strip in Cartesian coordinate system takes area $-\infty < x < \infty$, $0 \le y \le b$, $-h \le z \le h$.

In the equations of the motion of an elastic medium and in the equations of electrodynamics of Maxwell, with the use of the hypothesis of magnetoelasticity of

* E-mail: mbelubekyan@yahoo.com ** E-mail: aro088@mail.ru

thin bodies, for bending vibrations of plate the following equation are established (see [1, 2]).

$$\frac{\partial\psi}{\partial x}-\frac{\partial\varphi}{\partial y}+\frac{\mu}{c}\cdot\frac{\partial f}{\partial t}=0,\quad \frac{\partial f}{\partial y}-\frac{4\pi\sigma}{c}\varphi=\frac{h_2^+-h_2^-}{2h},$$
$$\frac{\partial f}{\partial x}+\frac{4\pi\sigma}{c}\left(\psi+\frac{\mu}{c}H_{01}\frac{\partial w}{\partial t}\right)=\frac{h_1^+-h_1^-}{2h}, \tag{1.1}$$

$$D\Delta^2 w-\frac{2h\mu\sigma}{c}\left(-H_{01}\psi-\frac{\mu}{c}H_{01}^2\frac{\partial w}{\partial t}-\frac{h^2}{3}\left(\frac{\partial}{\partial y}\left(H_{01}F\right)\right)\right)+2\rho h\frac{\partial^2 w}{\partial t^2}=0. \tag{1.2}$$

In the system of Eqs. (1.1), (1.2) ψ and φ are searched tangential components of induced electric field; f is the normal component of induced magnetic field; w is the searched deflection function of plate; h_1^+, h_1^-, h_2^+, h_2^- are the value of tangential component of induced magnetic field on the surface faces of plate $z=\pm h$. μ is the magnetic permeability of plate material; c is the constant equal speed of light in the vacuum. In the Eqs. (1.2) Δ^2 is the two-dimensional double Laplace operator, D is the flexural stiffness of plate.

$$D=\frac{2Eh^3}{3(1-\nu^2)},\ F=\frac{\partial\varphi}{\partial x}+\frac{\partial\psi}{\partial y}+\frac{\mu}{c}\cdot\frac{\partial}{\partial t}\left(H_{01}\frac{\partial w}{\partial y}\right),\ \Delta=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}. \tag{1.3}$$

The obtained system of Eqs (1.1), (1.2) is two-dimensional concerning searched functions w, φ, ψ and f (from x, y, t). However, these equations also contain searched values h_1^+, h_1^-, h_2^+, h_2^-. To finally reduce the problem of magnetic vibrations to two-dimensional, we propose searched values of tangential component of induced magnetic field on the surface faces of plate-strip to determine according to the model of perfect conductor.

According to the model of perfect conductor, for establishing vibrations of induced magnetic field, is determined by the following way:

$$\vec{h}=\mathrm{rot}\left(\vec{U}\times\vec{H}_0\right). \tag{1.4}$$

From (1.4) follows

$$h_1^+-h_1^-=2hH_{01}\frac{\partial^2 w}{\partial y^2},\ h_2^+-h_2^-=-2h\frac{\partial^2 w}{\partial x\partial y}H_{01}. \tag{1.5}$$

The settings of (1.5) in the system Eqs. (1.1), (1.2) gives

$$\frac{\partial\psi}{\partial x}-\frac{\partial\varphi}{\partial y}+\frac{\mu}{c}\cdot\frac{\partial f}{\partial t}=0,\quad \frac{\partial f}{\partial y}-\frac{4\pi\sigma}{c}\varphi+\frac{\partial^2 w}{\partial x\partial y}H_{01}=0,$$
$$\frac{\partial f}{\partial x}+\frac{4\pi\sigma}{c}\left(\psi+\frac{\mu}{c}H_{01}\frac{\partial w}{\partial t}\right)-\frac{\partial^2 w}{\partial y^2}H_{01}=0, \tag{1.6}$$
$$D\Delta^2 w-\frac{2h\mu\sigma}{c}\left(-H_{01}\psi-\frac{\mu}{c}H_{01}^2\frac{\partial w}{\partial t}-\frac{h^2}{3}\left(\frac{\partial}{\partial y}\left(H_{01}F\right)\right)\right)+2\rho h\frac{\partial^2 w}{\partial t^2}=0.$$

The systems (1.6) from the four equations concerning the four searched function component of electromagnetic field φ, ψ and f and searched deflection of plate-strip w, there is the result of data of three-dimensional problem of magnetoelastic vibration of plate-strip to two-dimensional. From the second and the third equation systems (1.6) defining the values of φ and ψ and putting the values in the first equations, the system equations from four is brought to the system from two equations:

$$\frac{\partial f}{\partial t}-\frac{c^2}{4\pi\mu\sigma}\Delta f=H_{01}\frac{\partial^2 w}{\partial x\partial t},$$

$$D\Delta^2 w+\frac{\mu h}{2\pi}H_{01}^2\frac{\partial^2 w}{\partial y^2}+\frac{\mu h^3}{6\pi}H_{01}^2\frac{\partial^2}{\partial y^2}\Delta w+2\rho h\frac{\partial^2 w}{\partial t^2}=\frac{\mu h}{2\pi}H_{01}\frac{\partial f}{\partial x}. \tag{1.7}$$

From the system (1.7) for the limiting case of perfect conductor $(\sigma\to\infty)$ is obtained next equation of bending vibrations of plate-strip:

$$D\Delta^2 w-\frac{\mu h H_{01}^2}{2\pi}\left(\frac{\partial^2 w}{\partial x^2}-\frac{\partial^2 w}{\partial y^2}\right)+\frac{\mu h^3 H_{01}^2}{6\pi}\cdot\frac{\partial^2}{\partial y^2}\Delta^2 w+2\rho h\frac{\partial^2 w}{\partial t^2}=0. \tag{1.8}$$

2. The Vibration of Plate-Strip. The solution of the system (1.8) for plate-strip $(-\infty<x<\infty)$ is proposed in the following way

$$w=w_0 e^{i(\omega t-kx)}. \tag{2.1}$$

Change of variables of (2.1) in (1.8) gives

$$(1+\chi)w_0^{IV}-2k^2\left(1-\frac{3\chi}{2k^2h^2}+\frac{\chi}{2}\right)w_0^{II}+k^4\left(1+\frac{3\chi}{k^2h^2}-\Omega^2\right)w_0=0, \tag{2.2}$$

where the following notes are done $\chi=\dfrac{\mu h^3 H_{01}^2}{6\pi D}$, $\Omega^2=\dfrac{2\rho h\omega^2}{Dk^4}$.

The solution of the system (1.7) for plate-strip ($-\infty<x<\infty$) is proposed by the following

$$w=w_0 e^{\gamma t-ikx},\quad f=f_0 e^{\gamma t-ikx}. \tag{2.3}$$

Change of variables of (2.3) in Eq. (1.7) gives

$$(1+\alpha k^2)f_0-\alpha f_0^{II}=-ikH_{01}w_0,$$

$$(1+\chi)w_0^{IV}-2k^2\left(1-\frac{3\chi}{2k^2h^2}+\frac{\chi}{2}\right)w_0^{II}+k^4\left(1+\Omega^2\right)w_0=-ik\frac{\mu h H_{01}}{2\pi D}f_0, \tag{2.4}$$

where $\alpha=\dfrac{c^2}{4\pi\sigma\mu\gamma}$, $\Omega^2=\dfrac{2\rho h\gamma^2}{Dk^4}$.

From the Eq. (2.4) for w_0, we will get

$$\begin{aligned}&\frac{\delta}{k^2h}(1+\chi)w_0^{VI}-\Omega\left[\left(1+\frac{\delta}{\Omega h}\right)(1+\chi)+2\frac{\delta}{\Omega h}\left(1-\frac{3\chi}{k^2h^2}+\frac{\chi}{2}\right)\right]w_0^{IV}+\\&+k^2\Omega\left[2\left(1+\frac{\delta}{\Omega h}\right)\left(1-\frac{3\chi}{2k^2h^2}+\frac{\chi}{2}\right)+\frac{\delta}{\Omega h}\left(1+\Omega^2\right)\right]w_0^{II}-\\&-\Omega k^4\left[\left(1+\frac{\delta}{\Omega h}\right)\left(1+\Omega^2\right)+\frac{3\chi}{k^2h^2}\right]w_0=0.\end{aligned} \tag{2.5}$$

where $\delta = \dfrac{\sqrt{3}c^2}{4\pi\sigma\mu a}$, $\dfrac{1}{a} = \sqrt{\dfrac{\rho\left(1-\nu^2\right)}{E}}$.

Looking at the bending vibrations of plate-strip for the hinge supported boundary case, which were written when

$$y = 0, b, \quad w = 0, \quad \frac{\partial^2 w}{\partial y^2} = 0, \quad f = 0. \tag{2.6}$$

The solution of the Eqs. (2.5) and (2.2) according to the boundary conditions (2.6) will be

$$w_0 = A_n \sin\left(p_n y\right), \ p_n = \frac{\pi n}{b}, \tag{2.7}$$

$$\begin{gathered}\Omega^3 + \left(1 + \frac{p_n^2}{k^2}\right)\frac{\delta}{h}\Omega^2 + \left[\frac{p_n^4}{k^4}(1+\chi) + 2\frac{p_n^2}{k^2}\left(1 - \frac{3\chi}{2k^2h^2} + \frac{\chi}{2}\right) + 1 + \frac{3\chi}{k^2h^2}\right]\Omega + \\ + \frac{\delta}{h}\left[\frac{p_n^6}{k^6}(1+\chi) + \frac{p_n^4}{k^4}\left(3 - \frac{3\chi}{k^2h^2} + 2\chi\right) + \frac{p_n^2}{k^2}\left(3 - \frac{3\chi}{k^2h^2} + \chi\right) + 1\right] = 0,\end{gathered} \tag{2.8}$$

$$\Omega^2 = \left[\frac{p_n^4}{k^4}(1+\chi) + 2\frac{p_n^2}{k^2}\left(1 - \frac{3\chi}{2k^2h^2} + \frac{\chi}{2}\right) + 1 + \frac{3\chi}{k^2h^2}\right]. \tag{2.9}$$

3. Numerical Results. Before giving numerical results in the Eqs. (2.8) and (2.9), we'll do the next assumptions, as $k^2h^2 << 1$, then we'll have

$$\begin{gathered}\Omega^3 + \left(1 + \frac{p_n^2}{k^2}\right)\frac{\delta}{h}\Omega^2 + \left[\frac{p_n^4}{k^4}(1+\chi) + 2\frac{p_n^2}{k^2}\left(1 - \frac{3\chi}{2k^2h^2}\right) + 1 + \frac{3\chi}{k^2h^2}\right]\Omega + \\ + \frac{\delta}{h}\left[\frac{p_n^6}{k^6}(1+\chi) + \frac{p_n^4}{k^4}\left(3 - \frac{3\chi}{k^2h^2}\right) + \frac{p_n^2}{k^2}\left(3 - \frac{3\chi}{k^2h^2}\right) + 1\right] = 0,\end{gathered} \tag{3.1}$$

$$\Omega^2 = \left[\frac{p_n^4}{k^4}(1+\chi) + 2\frac{p_n^2}{k^2}\left(1 - \frac{3\chi}{2k^2h^2}\right) + 1 + \frac{3\chi}{k^2h^2}\right]. \tag{3.2}$$

For Eqs. (3.1) and (3.2) numerical solutions were brought by following values $kh = \dfrac{1}{20}$, $\dfrac{p_n}{k} = \dfrac{1}{100}$, $\dfrac{\delta}{h} = \{0;\ 0.01;\ 0.05;\ 0.1\}$, χ is the dimensionless values characterize the value of magnetic field $\chi = \{0;\ 0.02; 0.04; 0.06;\ 0.08;\ 0.1\}$.

In the Tab. 1 were brought the values $\Omega_* = \Omega kh$ for the perfectly conductive plate-strip and $Im\,\Omega_*$ for conductive plate-strip, and in the Tab. 2 are brought the values of damping coefficients β .

Table 1

Vibration frequency

–	Ω_*	$Im\Omega_*$		
χ	–	$\frac{\delta}{h}=0.01$	$\frac{\delta}{h}=0.05$	$\frac{\delta}{h}=0.1$
0	0.050005	0.050005	0.050005	0.050005
0.02	0.249989	0.249989	0.249986	0.249976
0.04	0.349984	0.349983	0.349981	0.349974
0.06	0.42718	0.42718	0.427178	0.427172
0.08	0.492419	0.492419	0.492417	0.492413
0.1	0.549973	0.549973	0.549972	0.549967

Table 2

Damping coefficients

–	β damping coefficients		
χ	$\frac{\delta}{h}=0.01$	$\frac{\delta}{h}=0.05$	$\frac{\delta}{h}=0.1$
0	-5×10^{-5}	-2.50×10^{-3}	5×10^{-3}
0.02	$-1{,}99\times10^{-5}$	-9.98×10^{-5}	1.99×10^{-4}
0.04	-10^{-5}	-5.07×10^{-5}	10^{-4}
0.06	-6.80×10^{-6}	-3.40×10^{-5}	-6.80×10^{-5}
0.08	-5.11×10^{-6}	-2.55×10^{-5}	-5.11×10^{-5}
0.1	-4.08×10^{-6}	2.04×10^{-5}	-4.08×10^{-5}

According to the Tab. 1, the frequency values of vibrations for the perfectly conductive and electroconductive plate-strip do not differ. But, unlike perfectly conductive plate-strip, for electroconductive plate-strip are also obtained damping coefficients. Also, according to the Tab. 1, by increasing magnetic field the frequency vibrations are being increased. And in the Tab. 2 is shown that by increasing magnetic field the damping coefficient decreases.

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