PROCEEDINGS OF THE YEREVAN STATE UNIVERSITY

Physical and Mathematical Sciences 2020, **54**(3), p. 137–145

Mathematics

ON LOCALLY-BALANCED 2-PARTITIONS OF BIPARTITE GRAPHS

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A 2-*partition of a graph* G is a function $f: V(G) \to \{0,1\}$. A 2-partition f of a graph G is a *locally-balanced with an open neighborhood*, if for every $v \in V(G)$, $||\{u \in N_G(v): f(u)=0\}| - |\{u \in N_G(v): f(u)=1\}|| \leq 1$. A bipartite graph is (a,b)-*biregular* if all vertices in one part have degree a and all vertices in the other part have degree b. In this paper we prove that the problem of deciding, if a given graph has a locally-balanced 2-partition with an open neighborhood is NP-complete even for $(3,8)$-biregular bipartite graphs. We also prove that a $(2,2k+1)$-biregular bipartite graph has a locally-balanced 2-partition with an open neighbourhood if and only if it has no cycle of length $2 \pmod 4$. Next, we prove that if G is a subcubic bipartite graph that has no cycle of length 2 $\pmod 4$, then G has a locally-balanced 2-partition with an open neighbourhood. Finally, we show that all doubly convex bipartite graphs have a locally-balanced 2-partition with an open neighbourhood.

https://doi.org/10.46991/PYSU:A/2020.54.3.137

MSC2010: 05C70; 68Q25.

Keywords: locally-balanced 2-partition, NP-completeness, bipartite graph, biregular bipartite graph, subcubic bipartite graph.

Introduction. In this paper all graphs are finite, undirected, and have no loops or multiple edges, unless otherwise stated. Let $V(G)$ and $E(G)$ denote the sets of vertices and edges of a graph G, respectively. The set of neighbors of a vertex v in G is denoted by $N_G(v)$. The degree of a vertex $v \in V(G)$ is denoted by $d_G(v)$ and the maximum degree of vertices in G by $\Delta(G)$. A bipartite graph G with a bipartition (X,Y) is (a,b)-*biregular* if each vertex in X has degree a and each vertex in Y has degree b. A bipartite graph G with bipartition (X,Y) is *doubly convex*, if all its vertices from X can be numbered $1,2,\ldots,|X|$ and all vertices from Y can be numbered $1,2,\ldots,|Y|$ such that for every vertex of G the set of numbers assigned to neighbors is an interval of integers. The terms and concepts that we do not define can be found in [1,2].

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Graph partition problems play a crucial role in the current research. There are many different applications of graph partition problems (VLSI design, parallel computing, task scheduling, clustering and detection of communities in complex networks), and there are many problems in graph theory which can be formulated as graph partition problems (factorization problems, coloring problems, clustering problems, problems of Ramsey Theory). For example, the problem of finding the arboricity of a graph coinsides with the problem of decomposing of a graph into minimum number of forests, the problem of determining chromatic number of a graph is the problem of decomposing a graph into minimum number of independent sets, and the problem of determining chromatic index of a graph is equivalent to the decomposing of a graph into minimum number of matchings. Some other applications of graph partition problems can be found in [3].

The concept of locally-balanced 2-partition of graphs was introduced by Balikyan and Kamalian in 2005 [4]. Locally-balanced 2-partitions of graphs can be considered as a special case of equitable colorings of hypergraphs [5]. Berge [5] obtained some sufficient conditions for the existence of equitable colorings of hypergraphs. Ghouila-Houri [6] characterized unimodular hypergraphs in terms of partial equitable colorings and proved that a hypergraph $H=(V,E)$ is unimodular if and only if for each $V_0 \subseteq V$ there is a 2-coloring $\alpha : V_0 \rightarrow \{0,1\}$ such that for every $e \in E$, $\left||e \cap \alpha^{-1}(0)| - |e \cap \alpha^{-1}(1)|\right| \leq 1$. The problems of the existence and construction of a proper vertex-coloring of a graph, for which the number of vertices in any two color classes differ by at most one were considered in [7–10] . It was considered in [11], 2-vertex-colorings of graphs, for which each vertex is adjacent to the same number of vertices of every color. In particular, Kratochvil [11] proved that the problem of the existence of such a coloring is NP-complete even for the $(2p,2q)$-biregular $(p,q \geq 2)$ bipartite graphs. Moreover, he showed that the problem of existence of the aforementioned coloring for the $(2,2q)$-biregular $(q \geq 2)$ bipartite graphs can be solved in polynomial time. Gerber and Kobler [12, 13] suggested to consider the problem of deciding, if a given graph has a 2-partition with nonempty parts such that each vertex has at least as many neighbors in its part as in the other part. In [14], it was proved that the problem is NP-complete. In [4], Balikyan and Kamalian proved that the problem of existence of a locally-balanced 2-partition with an open neighborhood of bipartite graphs with maximum degree 3 is NP-complete. In 2006, a similar result for locally-balanced 2-partitions with a closed neighborhood was also proved [15]. In [16, 17], necessary and sufficient conditions for the existence of locally-balanced 2-partitions of trees were obtained. Balikyan [18] obtained the necessary and sufficient conditions for the existence of locally-balanced 2-partitions of bipartite cactus graphs. Gharibyan and Petrosyan [19] obtained necessary and sufficient conditions for the existence of locally-balanced 2-partitions of complete multipartite graphs. Recently, Gharibyan [20] studied locally-balanced 2-partitions of even and odd graphs. In particular, he gave necessary conditions for the existence of locally-balanced 2-partitions of these graphs.

In this paper we study locally-balanced 2-partitions with an open neighborhood of bipartite graphs. In particular, we prove that the problem of deciding, if a given graph has a locally-balanced 2-partition with an open neighborhood is *NP*-complete even for $(3,8)$-biregular bipartite graphs. We also show that a $(2,2k+1)$-biregular bipartite graph ($k \in \mathbb{N}$) has a locally-balanced 2-partition with an open neighbourhood if and only if it has no cycle of length $2 \pmod 4$. Next we prove that if G is a subcubic bipartite graph that has no cycle of length $2 \pmod 4$, then G has a locally-balanced 2-partition with an open neighbourhood. Finally, we show that all doubly convex bipartite graphs have a locally-balanced 2-partition with an open neighbourhood.

Main Results. Before we formulate and prove our main results, we introduce some terminology and notation. If G is a connected graph, then the distance between two vertices u and v in G will be denoted by $d_G(u,v)$. If φ is a 2-partition of a graph G and $v \in V(G)$, then define $\#(v)_\varphi$ and $\varphi^*(v)$ as follows:

$$\#(v)_\varphi = |\{u \in N_G(v) \colon \varphi(u) = 1\}| - |\{u \in N_G(v) \colon \varphi(u) = 0\}|,$$

$$\varphi^*(v) = \begin{cases} -1, & \text{if } \varphi(v) = 0, \\ 1, & \text{if } \varphi(v) = 1. \end{cases}$$

It is easy to see that

$$\#(v)_\varphi = \sum_{u \in N_G(v)} \varphi^*(u).$$

A spanning subgraph F of a graph G is called an *$[a,b]$-factor* of G, if $a \leq d_F(v) \leq b$ for all $v \in V(G)$. We will use the following result from the factor theory.

Theorem 1. *[21]. Let k and r be integers such that $1 \leq k < r$. Then every r-regular graph (where multiple edges and loops are allowed) has a $[k,k+1]$-factor.*

We denote by $V = \{x_1,\dots,x_n\}$ a finite set of variables. A literal is either a variable x or a negated variable $\bar{x}$. We denote by $L_V = \{x,\bar{x} : x \in V\}$ the set of literals. A *clause* is a set of literals, i.e. a subset of L_V, and a *k-clause* is one which contains exactly k distinct literals. A clause is *monotone*, if all of its involved variables contain no negations.

We define a function $NAE_n : \{0,1\}^n \to \{0,1\}$ in the following way:

$$NAE_n(x_1,x_2,\dots,x_n) = \begin{cases} 0, & \text{if } x_1 = x_2 = \dots = x_n, \\ 1, & \text{otherwise.} \end{cases}$$

If c is a monotone r-clause and $x_{i_1},x_{i_2},\dots,x_{i_r} \in c$, then define $NAE_r(c)$ as follows:

$$NAE_r(c) = NAE_r(x_{i_1},x_{i_2},\dots,x_{i_r}).$$

Let us consider the following

Problem 1 (*NAE-3-Sat-E4*).

Instance: given a set $V = \{x_1,\dots,x_n\}$ of variables and a collection $C = \{c_1,\dots,c_k\}$ of monotone 3-clauses over V such that every variable appears in exactly four clauses.

Question: is the formula $f(x_1,\dots,x_n) = NAE_3(c_1)\ \&\ \cdots\ \&\ NAE_3(c_k)$ satisfied?

The following was proved [22].

Theorem 2. *[22]. Problem 1 is NP-complete.*

Let us now consider the following

Problem 2.

Instance: a $(3,8)$-biregular bipartite graph G.

Question: does G has a locally-balanced 2-partition with an open neighbourhood?

Theorem 3. *Problem 2 is NP-complete.*

Proof. It is easy to see that Problem 2 is in NP. For the proof of the NP-completeness, we show a reduction from Problem 1 to Problem 2. Let $\mathcal{I} = (V,C)$ be an instance of Problem 1. We must construct a $(3,8)$-biregular bipartite graph $G = (X,Y;E)$ such that G has a locally-balanced 2-partition with an open neighbourhood if and only if $f(x_1,\dots,x_n) = NAE_3(c_1)\ \&\ \cdots\ \&\ NAE_3(c_k)$ formula is satisfiable. Let us construct a graph G in the following way:

$$X = \{p_1,\dots,p_n\},$$
$$Y = \left\{q_1^1,q_1^2,q_2^1,q_2^2,\dots,q_k^1,q_k^2\right\},$$
$$E = \left\{p_iq_j^1, p_iq_j^2 :\ 1 \leq i \leq n, 1 \leq j \leq k, x_i \in c_j\right\}.$$

It is not hard to see that the graph G can be constructed from V and C in polynomial time, as we used only $n+2k$ vertices. We first suppose that $(\beta_1,\dots,\beta_n)$ is a true assignment of $f(x_1,\dots,x_n)$. We show that G has a locally-balanced 2-partition with an open neighbourhood.

Let us define a 2-partition φ of G as follows: for every $w \in V(G)$, let

$$\varphi(w) = \begin{cases} \beta_i, & \text{if} \quad w = p_i, \quad \text{where} \quad 1 \leq i \leq n, \\ l \bmod 2, & \text{if} \quad w = q_j^l, \quad \text{where} \quad 1 \leq j \leq k, 1 \leq l \leq 2. \end{cases} \tag{1}$$

Let us show that φ is a locally-balanced 2-partition with an open neighbourhood. Let us consider the vertices of the part X. Let $p_i \in X$ $(1 \leq i \leq n)$, where $x_i \in c_{j_1}$, $x_i \in c_{j_2}$, $x_i \in c_{j_3}$, $x_i \in c_{j_4}$, $1 \leq j_1, j_2, j_3, j_4 \leq k$. From this and taking into account (1), we have

$$\#(p_i)_\varphi = \varphi^*\left(q_{j_1}^1\right) + \varphi^*\left(q_{j_1}^2\right) + \varphi^*\left(q_{j_2}^1\right) + \varphi^*\left(q_{j_2}^2\right)$$
$$+\varphi^*\left(q_{j_3}^1\right) + \varphi^*\left(q_{j_3}^2\right) + \varphi^*\left(q_{j_4}^1\right) + \varphi^*\left(q_{j_4}^2\right) = 0.$$

Let us now consider the vertices of the part Y. Let $q_j^l \in Y$ $(1 \leq j \leq k,\ 1 \leq l \leq 2)$, where $c_j = \{x_{i_1},x_{i_2},x_{i_3}\}$. This means $NAE_3(\beta_{i_1},\beta_{i_2},\beta_{i_3}) = 1$. From this we get that for $1 \leq j \leq k$, $1 \leq l \leq 2$,

$$|\#(q_j^l)_\varphi| = |\varphi^*(p_{i_1}) + \varphi^*(p_{i_2}) + \varphi^*(p_{i_3})| \leq 1.$$

Conversely, suppose that α is a locally-balanced 2-partition with an open neighbourhood of G. Let us define an assignment of $f(x_1,\dots,x_n)$ as follows: $x_i = \alpha(p_i)$ $(1 \leq i \leq n)$. Let $c_j \in C$ $(1 \leq j \leq k)$ and $c_j = \{x_{i_1},x_{i_2},x_{i_3}\}$. From this and taking into account that $\left|\#\left(q_j^1\right)_\alpha\right| \leq 1$ and $d_G\left(q_j^1\right) = 3$, we obtain

$$NAE_3(x_{i_1},x_{i_2},x_{i_3}) = 1,$$

which implies that $f(x_1,\dots,x_n)$ is satisfiable. □

Next, we consider locally-balanced 2-partitions with an open neighbourhood of $(2,2k+1)$-biregular bipartite graphs. Before we move on we need the following simple lemma.

Lemma. *If φ is a locally-balanced 2-partition with an open neighbourhood of a graph G, then for every $v \in V(G)$ with $d_G(v)=2$, we have $\varphi(u_1) \neq \varphi(u_2)$, where $vu_1, vu_2 \in E(G)$ $(u_1 \neq u_2)$.*

Proof. Suppose φ is a locally-balanced 2-partition with an open neighbourhood of G. Let us consider a vertex $v \in V(G)$, where $d_G(v)=2$. Then

$$\#(v)_\varphi = 0 = |\{u:\ u \in N_G(v),\ \varphi(u)=1\}| - |\{u:\ u \in N_G(v),\ \varphi(u)=0\}|,$$

hence $|\{u:\ u \in N_G(v),\ \varphi(u)=1\}| = |\{u:\ u \in N_G(v),\ \varphi(u)=0\}|$. This implies that $\varphi(u_1) \neq \varphi(u_2)$. □

Theorem 4. *A $(2,2k+1)$-biregular bipartite graph G $(k \in \mathbb{N})$ with bipartition (X,Y) has a locally-balanced 2-partition with an open neighbourhood if and only if it has no cycle of length $2 \pmod 4$.*

Proof. Without loss of generality we may assume that G is connected. Next, assume that $C = x_{i_1}, y_{j_1}, x_{i_2}, y_{j_2}, \ldots, x_{i_{2r+1}}, y_{j_{2r+1}}, x_{i_1}$ is a cycle of length of $4r+2$ $(r \geq 1)$, where $x_{i_l} \in X$ and $y_{i_l} \in Y$ $(1 \leq l \leq 2r+1)$. Suppose, to the contrary, that there exists a locally-balanced 2-partition with an open neighbourhood φ of G. By Lemma , we have

$$\varphi(y_{j_l}) \neq \varphi(y_{j_{(l \bmod (2r+1))+1}})\ (1 \leq l \leq 2r+1). \tag{2}$$

Hence

$$\varphi(y_{j_1}) = \varphi(y_{j_3}) = \ldots = \varphi(y_{j_{2r+1}}),$$

which contradicts (2).

Now suppose that G has no cycle of length $4r+2$ $(r \geq 1)$. Let $Y^{(2)} = \{\{y,y'\} : y,y' \in Y, y \neq y'\}$. We define a function $f : X \to Y^{(2)}$ as follows: for every $x \in X$, let

$$f(x) = uv, \text{ where } u \in N_G(x) \text{ and } v \in N_G(x).$$

Let us now construct a graph G' (where multiple edges are allowed) in the following way:

$$V(G') = Y,$$
$$E(G') \text{ contains all possible edges } f(x), \text{ where } x \in X.$$

It is easy to see that G' is a $(2k+1)$-regular graph (where multiple edges are allowed). By Theorem 1, G' has a $[k,k+1]$-factor H. Let $\overline{y} \in V(G')$ be a vertex. Now, let us define a 2-partition φ of G by two steps as follows:

1. For $x \in X$, let

$$\varphi(x) = \begin{cases} 1, & \text{if } f(x) \in E(H), \\ 0, & \text{otherwise.} \end{cases}$$

2. For $y \in Y$, let

$$\varphi(y) = \begin{cases} 1, & \text{if } \ d_G(y,\overline{y}) \bmod 4 = 0, \\ 0, & \text{if } \ d_G(y,\overline{y}) \bmod 4 = 2. \end{cases}$$

Let us show that φ is a locally-balanced 2-partition with an open neighbourhood. Let us first consider the vertices of Y. Clearly, for any $v \in V(G')$,

$$||\{u : uv \in E(H)\}| - |\{u : uv \notin E(H)\}|| \leq 1.$$

This implies that for any $y \in Y$,

$$\left|\#(y)_\varphi\right| \leq 1.$$

Let us now consider the vertices of X. Suppose, to the contrary, that there exists $x_0 \in X$, where $x_0y_{i_1}, x_0y_{i_2} \in E(G)$, $y_{i_1} \neq y_{i_2}$ such that $\varphi(y_{i_1}) = \varphi(y_{i_2})$. This implies that

$$d_G(\overline{y}, y_{i_1}) \bmod 4 = d_G(\overline{y}, y_{i_2}) \bmod 4. \tag{3}$$

Let P_1 be the shortest path between the vertices y_{i_1} and $\overline{y}$ with the length r_1 and P_2 be the shortest path between the vertices $\overline{y}$ and y_{i_2} with the length r_2. From this and taking into account (3), we obtain

$$(r_1 + r_2) \bmod 4 = 0.$$

Let us consider a closed walk x_0, P_1, P_2, x_0. The length of the closed walk is $r_1 + r_2 + 2 = 4p + 2$ $(p \geq 1)$. It can be shown (by induction on the length of the closed walk) that each such closed walk contains a cycle of length $4t + 2$ in G $(t \leq p)$, which is a contradiction. □

In [4], Balikyan and Kamalian proved that the problem of existence of locally-balanced 2-partition with an open neighborhood of subcubic bipartite graphs is NP-complete. Nevertheless, we are able to prove the following result.

Theorem 5. *If G is a subcubic bipartite graph that has no cycle of length* 2 (mod 4)*, then G has a locally-balanced* 2*-partition with an open neighbourhood.*

Proof. Let G be a subcubic bipartite graph with bipartition (X,Y) that has no cycle of length 2 (mod 4). Since G is bipartite, so we may construct independently a 2-partition of the part Y, then using the same technique, we may construct a 2-partition of the part X. Let us define a set $A \subseteq E(G)$ as follows: we take one edge incident to each vertex $x \in X$ with $d_G(x) = 3$.

Let us consider the graph $G' = G - A$. Clearly, the graph G' has no cycle of length 2 (mod 4). Let us define a 2-partition φ of G'. Let $y' \in Y$ be a vertex of a component C of G'. For every $y \in V(C) \cap Y$, let

$$\varphi(y) = \begin{cases} 1, & \text{if } d_{G'}(y,y') \bmod 4 = 0, \\ 0, & \text{if } d_{G'}(y,y') \bmod 4 = 2. \end{cases}$$

Since C has no cycle of length 2 (mod 4), we obtain that φ is a 2-partition of the component C of G'. Using the same method for partitioning the vertices of each other component of G', we define a 2-partition φ for the part Y of the whole graph G'.

Using the same technique, we construct a 2-partition of the part X.
Let us show that φ is a locally-balanced 2-partition with an open neighbourhood. We will show for vertices of the part X and by the same method it can be shown for vertices of the part Y. Let x be a vertex of the part X. We consider three cases.

Case 1: $d_G(x) = 1$.

Since $d_G(x) = 1$, we have $|\#(x)_\varphi| = 1$.

Case 2: $d_G(x) = 2$.

Let $xy_{i_1}, xy_{i_2} \in E(G)$. Clearly, there exists a component C' of G' such that $y_{i_1}, y_{i_2} \in V(C')$. Thus,

$$|\#(x)_\varphi| = 0.$$

Case 3: $d_G(x) = 3$.

Let $xy_{i_1}, xy_{i_2}, xy_{i_3} \in E(G)$ and $xy_{i_3} \in A$. Clearly, there exists a component C'' of G' such that $y_{i_1}, y_{i_2} \in V(C'')$. Thus,

$$|\#(x)_\varphi| = |\varphi^*(y_{i_3})| = 1.$$

□

Finally, we consider doubly convex bipartite graphs and show that all these have a locally-balanced 2-partition with an open neighbourhood.

Proposition. *If G is a doubly convex bipartite graph with bipartition (X,Y), then G has a locally-balanced 2-partition with an open neighbourhood.*

Proof. Let G be a doubly convex bipartite graph with bipartition (X,Y), where all its vertices from X are numbered $1,2,\dots,|X|$ and all vertices from Y are numbered $1,2,\dots,|Y|$, so that for every vertex of G, the set of numbers assigned to neighbors is an interval of integers.

Let us define a 2-partition φ of G as follows:

1) for every $x \in X$, let

$$\varphi(x) = \begin{cases} 1, & \text{if the number of } x \text{ is odd,} \\ 0, & \text{if the number of } x \text{ is even;} \end{cases}$$

2) for every $y \in Y$, let

$$\varphi(y) = \begin{cases} 1, & \text{if the number of } y \text{ is odd,} \\ 0, & \text{if the number of } y \text{ is even.} \end{cases}$$

It is easy to see that φ is a locally-balanced 2-partition with an open neighbourhood of G. □

Received 02.10.2020
Reviewed 01.12.2020
Accepted 15.12.2020

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ԵՐԿԿՈՂՄԱՆԻ ԳՐԱՖՆԵՐԻ ԼՈԿԱԼ-ՀԱՎԱՍԱՐԱԿՇՌՎԱԾ 2-ՏՐՈՀՈՒՄՆԵՐԻ ՄԱՍԻՆ

$f : V(G) \rightarrow \{0,1\}$ ֆունկցիան կոչվում է G *գրաֆի* 2-*տրոհում*: G գրաֆի f 2-տրոհումը կանվանենք *լոկալ-հավասարակշռված բաց շրջակայքով*, եթե կամայական $v \in V(G)$-ի համար $\big||\{u \in N_G(v) : f(u) = 0\}| - |\{u \in N_G(v) : f(u) = 1\}|\big| \leq 1$: Երկկողմանի գրաֆը կանվանենք (a,b)-*երկհամասեռ*, եթե մի կողմի բոլոր գագաթների աստիճանը a է, իսկ մյուս կողմի բոլոր գագաթներինը՝ b: Այս աշխատանքում ապացուցվում է, որ տրված գրաֆի լոկալ-հավասարակշռված 2-տրոհման գոյության խնդիրը բաց շրջակայքի դեպքում NP-լրիվ է նույնիսկ $(3,8)$-երկհամասեռ երկկողմանի գրաֆների համար: Աշխատանքում ցույց է տրվում նաև, որ $(2,2k+1)$-երկհամասեռ երկկողմանի գրաֆը ունի լոկալ-հավասարակշռված 2-տրոհում այն և միայն այն դեպքում, եթե այն չի պարունակում $2 \pmod 4$ երկարությամբ պարզ ցիկլ: Հաջորդիվ, ապացուցում է, որ ենթախորանարդ երկկողմանի գրաֆը, որը չի պարունակում $2 \pmod 4$ երկարությամբ պարզ ցիկլ, ունի լոկալ-հավասարակշռված 2-տրոհում բաց շրջակայքով: Աշխատանքի վերջում ցույց է տրվում, որ բոլոր երկակի ուռուցիկ երկկողմանի գրաֆները ունեն լոկալ-հավասարակշռված 2-տրոհում բաց շրջակայքով:

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О ЛОКАЛЬНО-СБАЛАНСИРОВАННЫХ 2-РАЗБИЕНИЯХ ДВУДОЛЬНЫХ ГРАФОВ

2-*Разбиением графа* G называется функция $f : V(G) \rightarrow \{0,1\}$. 2-Разбиение f графа G называется *локально-сбалансированным с открытой окрестностью*, если для любой вершины $v \in V(G)$, $\big||\{u \in N_G(v) : f(u) = 0\}| - |\{u \in N_G(v) : f(u) = 1\}|\big| \leq 1$. Двудольный граф называется (a,b)-*бирегулярным*, если все вершины одной доли имеют степень a, а все вершины другой доли имеют степень b. В настоящей работе доказано, что задача существования локально-сбалансированных 2-разбиений с открытой окрестностью NP-полна даже в случае $(3,8)$-бирегулярных двудольных графов. Также доказано, что $(2,2k+1)$-бирегулярный двудольный граф имеет локально-сбалансированное 2-разбиение с открытой окрестностью тогда и только тогда, когда он не содержит простой цикл длины $2 \pmod 4$. Кроме того, в работе доказано, что если субкубический двудольный граф G не содержит простых циклов длины $2 \pmod 4$, то он имеет локально-сбалансированное 2-разбиение с открытой окрестностью. В конце работы показано, что все двояковыпуклые двудольные графы имеют локально-сбалансированное 2-разбиение с открытой окрестностью.