

ON A PROBLEM FOR AN ELASTIC INFINITE SHEET STRENGTHENED
BY TWO PARALLEL STRINGERS WITH FINITE LENGTHS
THROUGH ADHESIVE SHEAR LAYERS

A. V. KEROPYAN

Chair of Mechanics, YSU, Armenia

The article considers the problem for an elastic infinite sheet (plate), which is strengthened on two parallel finite parts of its upper surface by two parallel finite stringers with different elastic properties. The parallel stringers are located asymmetrically with respect to the horizontal axis of the sheet and deform under the action of horizontal forces. The interaction between the infinite sheet and stringers takes place through thin elastic adhesive layers. The problem of determining unknown shear stresses acting between the infinite sheet and stringers is reduced to a system of Fredholm integral equations of second kind with respect to unknown functions, which are specified on two parallel finite intervals. It is shown that in the certain domain of the change of the characteristic parameters of the problem this system of integral equations in Banach space can be solved by the method of successive approximations. Particular cases are considered, the character and behaviour of unknown shear stresses are investigated.

<https://doi.org/10.46991/PYSU:A/2020.54.3.153>

MSC2010: 74M15.

Keywords: elastic infinite plate, infinite sheet, stringer, contact, adhesive shear layer, system of integral equations, operator equation.

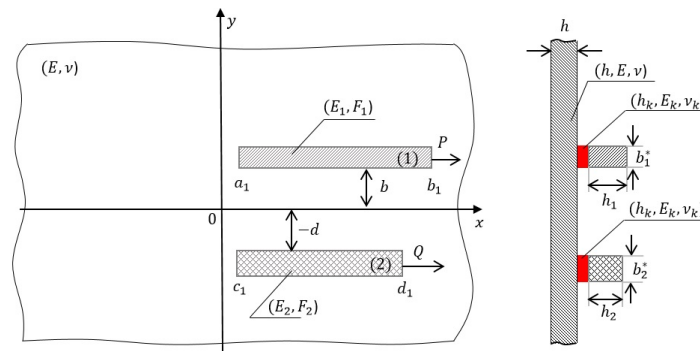
Introduction. The problems of loads transfer from two finite elastic stringers (overlays) to an elastic infinite strip and to an infinite sheet (or half-plane) through adhesive layers, when two stringers are arranged on the same line, with different approach to the solution are considered in [1, 2]. The paper [3] considers the problem for an infinite sheet with two finite stringers when only one of the stringers is connected through an adhesive layer. In [4–7], using various approaches, problems are investigated for various elastic bodies, which are strengthened by a single finite stringer through adhesive layer. In [8, 9], transfer of loads from a finite number of finite elastic stringers to an elastic infinite sheet (or half-plane) and to an infinite strip through adhesive layers is considered. Some contact problems for an elastic

* E-mail: agas50@ysu.am

infinite sheet strengthened by elastic stringers are considered in [10]. In this article, a problem is considered for an elastic infinite sheet, which is strengthened on two parallel finite parts of its upper surface by two parallel finite stringers having different elastic properties. The interaction between infinite sheet and stringers is assumed to be carried out through thin adhesive layers with different physical-mechanical properties and geometric configuration.

Statement of the Problem and Obtaining the System of Integral Equations.

Let an elastic infinite plate (sheet) of small constant thickness h , the Young's modulus E and the Poisson's ratio ν , which is in a generalized plane stress state (xOy is its middle plane), on its upper surface along $y = b$ ($b > 0$) and $y = -d$ ($d > 0$) parallel lines on the $[a_1, b_1]$ ($b_1 > a_1$) and $[c_1, d_1]$ ($d_1 > c_1$) finite intervals is strengthened by two finite stringers with modulus of elasticity equal to E_1 and E_2 , respectively. It is supposed that the stringers have a rectangular cross-sections with small areas $F_1 = b_1^* h_1$ and $F_2 = b_2^* h_2$, respectively, where b_1^* ($b_1^* \ll b_1 - a_1$), b_2^* ($b_2^* \ll d_1 - c_1$) are the widths of the stringers and h_1, h_2 are their small constant thicknesses. The interaction between infinite sheet and stringers is realized through thin, uniform, elastic adhesive layers with Young's modulus E_k , Poisson's ratio ν_k , and small constant thickness h_k . The problem is to specify the law of distribution of unknown stresses acting between the sheet and the stringers, when the concentrated forces P and Q are applied at one end points of the stringers $x = b_1$ and $x = d_1$, respectively, and are directed along the Ox axis (see Figure).



It is assumed that for the stringers the model of uniaxial strain state in combination with the model of contact along the line is realized [11, 12], and for the adhesive layers there are the pure shear conditions [4]. In the case of such assumptions, it is accepted that only tangential (shear) stresses act between sheet and stringers [1–9].

Taking into account the above assumptions [1–12], the differential equations for the equilibrium of the stringers on finite intervals $[a_1, b_1]$ and $[c_1, d_1]$ will be written in the following form:

$$\frac{d^2 u^{(1)}}{dx^2} = \frac{p(x)}{E_1 F_1}, \quad a_1 \leq x \leq b_1, \quad (1)$$

$$\frac{d^2 u^{(2)}}{dx^2} = \frac{q(x)}{E_2 F_2}, \quad c_1 \leq x \leq d_1, \quad (2)$$

with the following boundary conditions:

$$\left. \frac{du^{(1)}}{dx} \right|_{x=a_1} = 0, \quad \left. \frac{du^{(1)}}{dx} \right|_{x=b_1} = \frac{P}{E_1 F_1}, \quad (3)$$

$$\left. \frac{du^{(2)}}{dx} \right|_{x=c_1} = 0, \quad \left. \frac{du^{(2)}}{dx} \right|_{x=d_1} = \frac{Q}{E_2 F_2}. \quad (4)$$

Here $u^{(1)}(x) = u^{(1)}(x, b)$ and $u^{(2)}(x) = u^{(2)}(x, -d)$ are the horizontal displacements of the points of the stringers at $y = b$ and $y = -d$ parallel lines, on the $[a_1, b_1]$ and $[c_1, d_1]$ finite intervals, respectively, $p(x) = b_1^* \tau^{(1)}(x, b)$, $\tau^{(1)}(x, b)$ is the shear stresses, acting under of the stringer on the $[a_1, b_1]$ finite part, $q(x) = b_2^* \tau^{(2)}(x, -d)$, $\tau^{(2)}(x, -d)$ is the shear stresses, acting under of the stringer on the $[c_1, d_1]$ finite part.

On the other hand, in view of above assumptions, let write the horizontal displacements $u_1(x, b)$ and $u_2(x, -d)$ of the points of the elastic infinite plate (sheet), when tangential (shear) forces with intensity $p(x)$ and $q(x)$ act on the $[a_1, b_1]$ and $[c_1, d_1]$ finite intervals of its upper surface along $y = b$ and $y = -d$ parallel lines, respectively, in the form:

$$u_1(x, b) = \frac{1}{\pi A^*} \int_{a_1}^{b_1} \left(\ln \frac{1}{|x-s|} + C \right) p(s) ds + \frac{1}{\pi A^*} \int_{c_1}^{d_1} (N(x-s) + C) q(s) ds, \quad (5)$$

$$a_1 \leq x \leq b_1,$$

$$u_2(x, -d) = \frac{1}{\pi A^*} \int_{c_1}^{d_1} \left(\ln \frac{1}{|x-s|} + C \right) q(s) ds + \frac{1}{\pi A^*} \int_{a_1}^{b_1} (N(x-s) + C) p(s) ds, \quad (6)$$

$$c_1 \leq x \leq d_1,$$

where

$$N(x) = \ln \frac{1}{\sqrt{x^2 + (b+d)^2}} - \frac{\kappa(b+d)^2}{x^2 + (b+d)^2}, \quad (7)$$

$$A^* = \frac{4Eh}{(1+\nu)(3-\nu)}, \quad \kappa = \frac{1+\nu}{3-\nu},$$

C is arbitrary constant.

Without going into details, we only note that the horizontal displacement $u(x, y)$ of the points of an infinite sheet, arising in the upper half-plane, when tangential forces act on its surface along the line $y = -d$ ($d > 0$) with intensity $\tau(x)$ ($-\infty < x < \infty$), is given by the formula:

$$u(x, y) = \frac{1}{\pi A^*} \int_{-\infty}^{\infty} \left[\ln \frac{1}{\sqrt{(x-s)^2 + (y+d)^2}} - \frac{\kappa(y+d)^2}{(x-s)^2 + (y+d)^2} \right] \tau(s) ds + \text{const},$$

$$-\infty < x < \infty, \quad 0 < y < \infty.$$

Now, assuming that each differential element of the adhesive layers is in a condition of pure shear [1–9], the following contact conditions are obtained:

$$u^{(1)}(x) - u_1(x, b) = k_1^* p(x), \quad a_1 \leq x \leq b_1, \quad (8)$$

$$u^{(2)}(x) - u_2(x, -d) = k_2^* q(x), \quad c_1 \leq x \leq d_1, \quad (9)$$

where $k_j^* = h_k/b_j^*G_k$, $j = 1, 2$, $G_k = E_k/2(1 + \nu_k)$, G_k is the shear modulus of adhesive layers, $p(x) = b_1^*\tau^{(1)}(x, b) = b_1^*G_k\gamma_k^{(1)}(x, b)$, $q(x) = b_2^*\tau^{(2)}(x, -d) = b_2^*G_k\gamma_k^{(2)}(x, -d)$ and $\gamma_k^{(1)}(x, b)$, $\gamma_k^{(2)}(x, -d)$ are the shear deformations of the adhesive layers, on the $[a_1, b_1]$ and $[c_1, d_1]$ finite intervals, respectively.

Further, by virtue of (8) and (9), equations (1) and (2) can be written in the form:

$$\frac{d^2u^{(1)}}{dx^2} - \gamma^2u^{(1)}(x) = -\gamma^2u_1(x, b), \quad a_1 \leq x \leq b_1, \quad (10)$$

$$\frac{d^2u^{(2)}}{dx^2} - \alpha^2u^{(2)}(x) = -\alpha^2u_2(x, -d), \quad c_1 \leq x \leq d_1, \quad (11)$$

with the boundary conditions (3) and (4), respectively.

Here $\gamma^2 = 1/k_1^*E_1F_1$, $\alpha^2 = 1/k_2^*E_2F_2$.

The solutions to the boundary value problems (10) and (3), (11) and (4) we obtain in the form:

$$u^{(1)}(x) = u_0^{(1)}(x) + \gamma^2 \int_{a_1}^{b_1} G(x, s) u_1(s, b) ds, \quad a_1 \leq x \leq b_1, \quad (12)$$

$$u^{(2)}(x) = u_0^{(2)}(x) + \alpha^2 \int_{c_1}^{d_1} K(x, s) u_2(s, -d) ds, \quad c_1 \leq x \leq d_1, \quad (13)$$

where $u_0^{(1)}(x)$ and $u_0^{(2)}(x)$ are the general solutions of the homogenous equations corresponding to equations (10) and (11), respectively, with the boundary conditions (3) and (4), respectively, and have the following form:

$$u_0^{(1)}(x) = \frac{P \cosh[\gamma(x - a_1)]}{\gamma E_1 F_1 \sinh[\gamma(b_1 - a_1)]}, \quad u_0^{(2)}(x) = \frac{Q \cosh[\alpha(x - c_1)]}{\alpha E_2 F_2 \sinh[\alpha(d_1 - c_1)]}.$$

In equations (12) and (13)

$$u_*^{(1)}(x) = \gamma^2 \int_{a_1}^{b_1} G(x, s) u_1(s, b) ds \quad \text{and} \quad u_*^{(2)}(x) = \alpha^2 \int_{c_1}^{d_1} K(x, s) u_2(s, -d) ds$$

are the particular solutions of (10) and (11), with zero boundary conditions corresponding to conditions (3) and (4), respectively, $G(x, s)$ and $K(x, s)$ are Green's functions:

$$G(x, s) = \frac{1}{\gamma \sinh[\gamma(b_1 - a_1)]} \begin{cases} \cosh[\gamma(x - b_1)] \cosh[\gamma(s - a_1)], & x > s, \\ \cosh[\gamma(x - a_1)] \cosh[\gamma(s - b_1)], & x < s; \end{cases}$$

$$K(x, s) = \frac{1}{\alpha \sinh[\alpha(d_1 - c_1)]} \begin{cases} \cosh[\alpha(x - d_1)] \cosh[\alpha(s - c_1)], & x > s, \\ \cosh[\alpha(x - c_1)] \cosh[\alpha(s - d_1)], & x < s. \end{cases}$$

It is obvious, that the functions $G(x, s)$ and $K(x, s)$ are continuous functions and $G(x, s) = G(s, x)$ and $K(x, s) = K(s, x)$.

Further, by virtue of (12) and (13), according to (8) and (9), we obtain the following equations:

$$k_1^* p(x) + u_1(x, b) = \gamma^2 \int_{a_1}^{b_1} G(x, s) u_1(s, b) ds + u_0^{(1)}(x), \quad a_1 \leq x \leq b_1, \quad (14)$$

$$k_2^* q(x) + u_2(x, -d) = \alpha^2 \int_{c_1}^{d_1} K(x, s) u_2(s, -d) ds + u_0^{(2)}(x), \quad c_1 \leq x \leq d_1. \quad (15)$$

Now, by virtue of (5) and (6), from (14) and (15), we obtain the following system of integral equations:

$$\begin{aligned} p(x) + \frac{1}{\pi A^* k_1^*} \left[\int_{a_1}^{b_1} \left(\ln \frac{1}{|x-s|} + C \right) p(s) ds + \int_{c_1}^{d_1} (N(x-s) + C) q(s) ds \right] = \\ = \frac{\gamma^2}{\pi A^* k_1^*} \int_{a_1}^{b_1} G(x, s) \left[\int_{a_1}^{b_1} \left(\ln \frac{1}{|s-t|} + C \right) p(t) dt + \int_{c_1}^{d_1} (N(s-t) + C) q(t) dt \right] ds + \\ + \frac{u_0^{(1)}(x)}{k_1^*}, \quad a_1 \leq x \leq b_1, \quad (16) \end{aligned}$$

$$\begin{aligned} q(x) + \frac{1}{\pi A^* k_2^*} \left[\int_{c_1}^{d_1} \left(\ln \frac{1}{|x-s|} + C \right) q(s) ds + \int_{a_1}^{b_1} (N(x-s) + C) p(s) ds \right] = \\ = \frac{\alpha^2}{\pi A^* k_2^*} \int_{c_1}^{d_1} K(x, s) \left[\int_{c_1}^{d_1} \left(\ln \frac{1}{|s-t|} + C \right) q(t) dt + \int_{a_1}^{b_1} (N(s-t) + C) p(t) dt \right] ds + \\ + \frac{u_0^{(2)}(x)}{k_2^*}, \quad c_1 \leq x \leq d_1. \end{aligned}$$

It should be noted that the spectrum of the symmetric second-order differential operator $D = -d^2/dx^2 + \gamma^2 I$ with the domain of definition being twice continuous differentiating functions, satisfying the boundary conditions $(du^{(1)}/dx)_{x=a} = 0$ and $(du^{(1)}/dx)_{x=b} = 0$, are eigenvalues $\lambda_n = \gamma^2 + n^2 \pi^2 / (b-a)^2$ ($n = 0, 1, 2, \dots$) with corresponding eigenfunctions $\cos[n\pi(x-a)/(b-a)]$ ($n = 0, 1, 2, \dots$).

It is known [13], that symmetric quite continuous integral operator B :

$$B\varphi = \int_a^b G(x, s) \varphi(s) ds,$$

which acts in the space $L_2(a, b)$ is an inverse of the operator D .

Hence, we have:

$$\int_{a_1}^{b_1} G(x, s) \cos \left[\frac{n\pi(s-a_1)}{b_1-a_1} \right] ds = \frac{(b_1-a_1)^2}{(b_1-a_1)^2 \gamma^2 + n^2 \pi^2} \cos \left[\frac{n\pi(x-a_1)}{b_1-a_1} \right], \quad (17)$$

$n = 0, 1, 2, \dots$

$$\int_{c_1}^{d_1} K(x, s) \cos \left[\frac{n\pi(s - c_1)}{d_1 - c_1} \right] ds = \frac{(d_1 - c_1)^2}{(d_1 - c_1)^2 \alpha^2 + n^2 \pi^2} \cos \left[\frac{n\pi(x - c_1)}{d_1 - c_1} \right], \quad (18)$$

$$n = 0, 1, 2, \dots,$$

where the functions $\cos \left[\frac{n\pi(x - a_1)}{b_1 - a_1} \right]$ ($n = 0, 1, 2, \dots$) and $\cos \left[\frac{n\pi(x - c_1)}{d_1 - c_1} \right]$ ($n = 0, 1, 2, \dots$) form full orthogonal systems in the spaces $L_2(a_1, b_1)$ and $L_2(c_1, d_1)$, respectively.

Further, after replacing the variables x, s and t by ax, as and at , respectively, where $a > 0$ is the coordinate of one of the end points of stringers, we get the system (16) as follows:

$$\begin{aligned} \varphi_1(x) + \bar{\delta}_1^2 \int_{\alpha_1}^{\beta_1} \ln \frac{1}{|x - t|} \varphi_1(t) dt - a\gamma^2 \bar{\delta}_1^2 \int_{\alpha_1}^{\beta_1} \left(\int_{\alpha_1}^{\beta_1} G(ax, as) \ln \frac{1}{|s - t|} ds \right) \varphi_1(t) dt + \\ + \bar{\delta}_1^2 \int_{\xi_1}^{\eta_1} N_1(x - t) \varphi_2(t) dt - a\gamma^2 \bar{\delta}_1^2 \int_{\xi_1}^{\eta_1} \left(\int_{\alpha_1}^{\beta_1} G(ax, as) N_1(s - t) ds \right) \varphi_2(t) dt - \\ - \frac{au_0^{(1)}(ax)}{k_1^*} = 0, \quad \alpha_1 \leq x \leq \beta_1, \quad (19) \end{aligned}$$

$$\begin{aligned} \varphi_2(x) + \bar{\delta}_2^2 \int_{\xi_1}^{\eta_1} \ln \frac{1}{|x - t|} \varphi_2(t) dt - a\alpha^2 \bar{\delta}_2^2 \int_{\xi_1}^{\eta_1} \left(\int_{\xi_1}^{\eta_1} K(ax, as) \ln \frac{1}{|s - t|} ds \right) \varphi_2(t) dt + \\ + \bar{\delta}_2^2 \int_{\alpha_1}^{\beta_1} N_1(x - t) \varphi_1(t) dt - a\alpha^2 \bar{\delta}_2^2 \int_{\alpha_1}^{\beta_1} \left(\int_{\xi_1}^{\eta_1} K(ax, as) N_1(s - t) ds \right) \varphi_1(t) dt - \\ - \frac{au_0^{(2)}(ax)}{k_2^*} = 0, \quad \xi_1 \leq x \leq \eta_1, \end{aligned}$$

since, according to (17) and (18), we have also the following equalities

$$\int_{\alpha_1}^{\beta_1} G(ax, as) ds = \frac{1}{a\gamma^2}, \quad \int_{\xi_1}^{\eta_1} K(ax, as) ds = \frac{1}{a\alpha^2}. \quad (20)$$

Here

$$\begin{aligned} \varphi_1(x) = ap(ax), \quad \varphi_2(x) = aq(ax), \quad \bar{\delta}_j^2 = a/\pi k_j^* A^*, \quad j = 1, 2, \\ N(ax) = \ln \frac{1}{a} + N_1(x), \quad N_1(x) = \ln \frac{1}{\sqrt{x^2 + (b_* + d_*)^2}} - \frac{\kappa(b_* + d_*)^2}{x^2 + (b_* + d_*)^2} \\ \alpha_1 = a_1/a, \quad \beta_1 = b_1/a, \quad \xi_1 = c_1/a, \quad \eta_1 = d_1/a, \quad b_* = b/a, \quad d_* = d/a. \end{aligned}$$

One can represent the system of integral equations (19) in the following form:

$$\begin{aligned} \varphi_1(x) + \bar{\delta}_1^2 \int_{\alpha_1}^{\beta_1} H_1(x,t) \varphi_1(t) dt + \bar{\delta}_1^2 \int_{\xi_1}^{\eta_1} H_2(x,t) \varphi_2(t) dt &= f_0(x), \quad \alpha_1 \leq x \leq \beta_1, \\ \varphi_2(x) + \bar{\delta}_2^2 \int_{\xi_1}^{\eta_1} M_1(x,t) \varphi_2(t) dt + \bar{\delta}_2^2 \int_{\alpha_1}^{\beta_1} M_2(x,t) \varphi_1(t) dt &= q_0(x), \quad \xi_1 \leq x \leq \eta_1, \end{aligned} \tag{21}$$

where

$$\begin{aligned} H_1(x,t) &= \ln \frac{1}{|x-t|} - a\gamma^2 \int_{\alpha_1}^{\beta_1} G(ax,as) \ln \frac{1}{|s-t|} ds, \\ H_2(x,t) &= N_1(x-t) - a\gamma^2 \int_{\alpha_1}^{\beta_1} G(ax,as) N_1(s-t) ds, \\ M_1(x,t) &= \ln \frac{1}{|x-t|} - a\alpha^2 \int_{\xi_1}^{\eta_1} K(ax,as) \ln \frac{1}{|s-t|} ds, \\ M_2(x,t) &= N_1(x-t) - a\alpha^2 \int_{\xi_1}^{\eta_1} K(ax,as) N_1(s-t) ds, \\ f_0(x) &= \frac{au_0^{(1)}(ax)}{k_1^*} = \frac{Pa\gamma \cosh[a\gamma(x-\alpha_1)]}{\sinh[a\gamma(\beta_1-\alpha_1)]}, \\ q_0(x) &= \frac{au_0^{(2)}(ax)}{k_2^*} = \frac{Qa\alpha \cosh[a\alpha(x-\xi_1)]}{\sinh[a\alpha(\eta_1-\xi_1)]}. \end{aligned}$$

It is obvious that the functions $f_0(x)$ and $q_0(x)$, are integrable functions on the segments $x \in [\alpha_1, \beta_1]$ and $x \in [\xi_1, \eta_1]$, respectively.

Note that the system of integral equations (19) or (21) are obtained by changing the order of integration, the validity of which follows from the Fubini's theorem [13]. This theorem will often be used below without special mention.

Now let us consider several particular cases that are directly obtained from the system of integral equations (21). In the case $\bar{\delta}_1^2 = \bar{\delta}_2^2 = 0$, from system (21) we obtain the solution of the corresponding problem for the case of a rigid sheet (i.e when $E \rightarrow \infty$) in the form $\varphi_1(x) = f_0(x)$, $x \in [\alpha_1, \beta_1]$ and $\varphi_2(x) = q_0(x)$ $x \in [\xi_1, \eta_1]$. In the case of one finite stringer defined on the segment $[a_1, b_1]$ or on the segment $[c_1, d_1]$, instead of system (21), we will obtain the Fredholm integral equation of the second kind with respect to an unknown function $\varphi_1(x)$ defined on the segment $[\alpha_1, \beta_1]$ in the following form:

$$\varphi_1(x) + \bar{\delta}_1^2 \int_{\alpha_1}^{\beta_1} H_1(x,t) \varphi_1(t) dt = f_0(x), \quad \alpha_1 \leq x \leq \beta_1, \tag{22}$$

or with respect to an unknown function $\varphi_2(x)$ defined on the segment $[\xi_1, \eta_1]$ in the form

$$\varphi_2(x) + \bar{\delta}_2^2 \int_{\xi_1}^{\eta_1} M_1(x, t) \varphi_2(t) dt = q_0(x), \quad \xi_1 \leq x \leq \eta_1 \quad (23)$$

respectively.

Note that the system of integral equations (21) was obtained without using the stringers equilibrium conditions:

$$\int_{\alpha_1}^{\beta_1} p(ax) dx = P/a, \quad \int_{\xi_1}^{\eta_1} q(ax) dx = Q/a. \quad (24)$$

In the system (21), the conditions (24) are satisfied automatically, since the following equalities hold:

$$\int_{\alpha_1}^{\beta_1} f_0(x) dx = P, \quad \int_{\xi_1}^{\eta_1} q_0(x) dx = Q.$$

These can be easily verified by integrating the first equation of (21) from α_1 to β_1 and the second equation from ξ_1 to η_1 , then changing the order of integration in the resulting double integrals and taking into account the equalities

$$\int_{\alpha_1}^{\beta_1} H_1(x, t) dx = 0, \quad \int_{\alpha_1}^{\beta_1} H_2(x, t) dx = 0, \quad \int_{\xi_1}^{\eta_1} M_1(x, t) dx = 0, \quad \int_{\xi_1}^{\eta_1} M_2(x, t) dx = 0,$$

which follow from (20).

Thus, solving the problem is reduced to solving the system (21) of Fredholm integral equations of the second kind with squarely integrable kernels in two variables and with right-hand sides, which are the solutions of the problem in the case of rigid sheet. From the system (21) it is easy to see that at the end points of the stringers $x = \alpha_1$, $x = \beta_1$ and $x = \xi_1$, $x = \eta_1$, the values of $\varphi_1(x)$ and $\varphi_2(x)$, respectively, are finite.

Investigation of the Solvability of the System of Integral Equations (21).

Now write the system (21) in the following form

$$\varphi + S\varphi = g_0, \quad (25)$$

where

$$\varphi = \begin{pmatrix} \varphi_1(x) \\ \varphi_2(x) \end{pmatrix}, \quad g_0 = \begin{pmatrix} f_0(x) \\ q_0(x) \end{pmatrix}, \quad S = \begin{pmatrix} \bar{\delta}_1^2 s_{11} & \bar{\delta}_1^2 s_{12} \\ \bar{\delta}_2^2 s_{21} & \bar{\delta}_2^2 s_{22} \end{pmatrix},$$

$$s_{11} \varphi_1 = \int_{\alpha_1}^{\beta_1} H_1(x, t) \varphi_1(t) dt, \quad s_{12} \varphi_2 = \int_{\xi_1}^{\eta_1} H_2(x, t) \varphi_2(t) dt,$$

$$s_{21} \varphi_1 = \int_{\alpha_1}^{\beta_1} M_2(x, t) \varphi_1(t) dt, \quad s_{22} \varphi_2 = \int_{\xi_1}^{\eta_1} M_1(x, t) \varphi_2(t) dt. \quad (26)$$

Further, consider operator equation (25) in Banach space with elements $X = \begin{pmatrix} x_1(z) \\ x_2(z) \end{pmatrix}$, where $x_1(z) \in L_2(\alpha_1, \beta_1)$, $x_2(z) \in L_2(\xi_1, \eta_1)$, and with norm:

$$\|X\| = \max \left\{ \|x_1(z)\|_{L_2(\alpha_1, \beta_1)}, \|x_2(z)\|_{L_2(\xi_1, \eta_1)} \right\}.$$

L_2 is a space of square integrable functions.

Operators s_{11} and s_{22} act in the spaces $L_2(\alpha_1, \beta_1)$, $L_2(\xi_1, \eta_1)$ respectively, and operators s_{12} and s_{21} act in the following form: $s_{12} : L_2(\xi_1, \eta_1) \rightarrow L_2(\alpha_1, \beta_1)$, $s_{21} : L_2(\alpha_1, \beta_1) \rightarrow L_2(\xi_1, \eta_1)$.

Obviously, the operator S acts in the B space and is a Fredholm operator. A sufficient condition for inversion of operator $I + S$ is the condition $\|S\| < 1$. In this case, the solution of operator equation (25) is written in the form

$$\varphi = (I + S)^{-1} g_0 = \sum_{m=0}^{\infty} (-1)^m S^m g_0.$$

Now let's determine the values of parameters of the problem, for which the condition $\|S\| < 1$ is satisfied, where

$$\|S\| = \max \left\{ \bar{\delta}_1^2 (\|s_{11}\| + \|s_{12}\|), \bar{\delta}_2^2 (\|s_{21}\| + \|s_{22}\|) \right\}.$$

Therefore, the condition $\|S\| < 1$ will be satisfied, if

$$\bar{\delta}_1^2 (\|s_{11}\| + \|s_{12}\|) < 1, \quad \bar{\delta}_2^2 (\|s_{21}\| + \|s_{22}\|) < 1. \tag{27}$$

From (26), by virtue of Cauchy–Bunyakovski inequality, we get:

$$\begin{aligned} \|s_{11}\| &\leq c_{11}, \quad c_{11} = \left(\int_{\alpha_1}^{\beta_1} \int_{\alpha_1}^{\beta_1} H_1^2(x, t) dx dt \right)^{\frac{1}{2}}, \\ \|s_{12}\| &\leq c_{12}, \quad c_{12} = \left(\int_{\xi_1}^{\eta_1} \int_{\alpha_1}^{\beta_1} H_2^2(x, t) dx dt \right)^{\frac{1}{2}}, \\ \|s_{21}\| &\leq c_{21}, \quad c_{21} = \left(\int_{\alpha_1}^{\beta_1} \int_{\xi_1}^{\eta_1} M_2^2(x, t) dx dt \right)^{\frac{1}{2}}, \\ \|s_{22}\| &\leq c_{22}, \quad c_{22} = \left(\int_{\xi_1}^{\eta_1} \int_{\xi_1}^{\eta_1} M_1^2(x, t) dx dt \right)^{\frac{1}{2}}. \end{aligned} \tag{28}$$

Obviously, the expressions for c_{ij} ($i, j = 1, 2$) are difficult to calculate, but they can be estimated. It was found out in [9] that the following estimates take place:

$$\begin{aligned} c_{11} &< \left(\int_{\alpha_1}^{\beta_1} \int_{\alpha_1}^{\beta_1} \ln^2|x-t| dx dt \right)^{\frac{1}{2}}, \quad c_{22} < \left(\int_{\xi_1}^{\eta_1} \int_{\xi_1}^{\eta_1} \ln^2|x-t| dx dt \right)^{\frac{1}{2}}, \\ c_{12} &< \left(\int_{\xi_1}^{\eta_1} \int_{\alpha_1}^{\beta_1} N_1^2(x-t) dx dt \right)^{\frac{1}{2}}, \quad c_{21} < \left(\int_{\alpha_1}^{\beta_1} \int_{\xi_1}^{\eta_1} N_1^2(x-t) dx dt \right)^{\frac{1}{2}}, \end{aligned} \tag{29}$$

The estimates (29) for c_{12} and c_{21} can be obtained also in the form

$$c_{12} < \frac{1}{2} \left(\int_{\xi_1}^{\eta_1} \int_{\alpha_1}^{\beta_1} \ln^2 [(x-t)^2 + (b_* + d_*)^2] dx dt \right)^{\frac{1}{2}} +$$

$$+ \kappa (b_* + d_*)^2 \left(\int_{\xi_1}^{\eta_1} \int_{\alpha_1}^{\beta_1} [(x-t)^2 + (b_* + d_*)^2]^{-2} dx dt \right)^{\frac{1}{2}},$$

$$c_{21} < \frac{1}{2} \left(\int_{\alpha_1}^{\beta_1} \int_{\xi_1}^{\eta_1} \ln^2 [(x-t)^2 + (b_* + d_*)^2] dx dt \right)^{\frac{1}{2}} +$$

$$+ \kappa (b_* + d_*)^2 \left(\int_{\alpha_1}^{\beta_1} \int_{\xi_1}^{\eta_1} [(x-t)^2 + (b_* + d_*)^2]^{-2} dx dt \right)^{\frac{1}{2}}.$$

Then the conditions (27) will be realized, if

$$\bar{\delta}_1^2 < (c_{11} + c_{12})^{-1} = c_1, \quad \bar{\delta}_2^2 < (c_{21} + c_{22})^{-1} = c_2,$$

where c_1 and c_2 are positive numbers less than unity.

Note also that from the condition of solvability of the integral equations (22) and (23), we obtain the following estimates for the parameters $\bar{\delta}_1^2$ and $\bar{\delta}_2^2$:

$$\bar{\delta}_1^2 < \frac{2}{l_1} \left(\int_{\alpha_1}^{\beta_1} \int_{\alpha_1}^{\beta_1} \ln^2 |x-t| dx dt \right)^{-\frac{1}{2}} \quad \text{and} \quad \bar{\delta}_2^2 < \frac{2}{l_2} \left(\int_{\xi_1}^{\eta_1} \int_{\xi_1}^{\eta_1} \ln^2 |x-t| dx dt \right)^{-\frac{1}{2}},$$

respectively.

The values of unknown shear stresses $\varphi_1(x)$ and $\varphi_2(x)$ at the end points $x = \alpha_1$, $x = \beta_1$ and $x = \xi_1$, $x = \eta_1$ of stringers, respectively, can be obtained from system (21).

Conclusion. Thus, in this article, we have presented an effective solution to the problem of changing the distribution law of unknown shear stresses specified on two parallel finite intervals. The problem is reduced to solving a system of Fredholm integral equations of the second kind with right-hand sides being the solutions of the problem in the case of a rigid sheet.

Note that the solution of the problem under consideration can be also reduced to solving a system of singular integro-differential equations of the second kind with Cauchy kernels and the corresponding boundary conditions. Its solutions can be constructed using Chebishev's orthogonal polynomials of the second kind [1].

Received 31.08.2020

Reviewed 03.11.2020

Accepted 18.11.2020

REFERENCES

1. Kerobyan A.V. Contact Problems for an Elastic Strip and the Infinite Plate with Two Finite Elastic Overlays in the Presence of Shear Interlayers. *Proc. NAS RA, Mechanics* **67** : 1 (2014), 22–34 (in Russian).
2. Kerobyan A.V. About Contact Problems for an Elastic Half-Plane and the Infinite Plate with Two Finite Elastic Overlays In the Presence of Shear Interlayers. *Proc. YSU A, Phys. Math. Sci.* **2** (2015), 30–38.
3. Grigoryan E.Kh., Kerobyan A.V., Shahinyan S.S. The Contact Problem for the Infinite Plate with Two Finite Stringers One from which is Glued, Other is Ideal Conducted. *Proc. NAS RA, Mechanics* **55** : 2 (2002), 14–23 (in Russian).
4. Lubkin J.L., Lewis L.C. Adhesive Shear Flow for an Axially Loaded, Finite Stringer Bonded to an Infinite Sheet. *Quart. J. Mech. Appl. Math.* **23** (1970), 521–533.
<https://doi.org/10.1093/qjmam/23.4.521>
5. Kerobyan A.V., Sarkisyan V.S. The Solution of the Problem for an Anisotropic Half-plane on the boundary of which Finite Length Stringer is Glued. In: *Proceedings of the Scientific Conference, Dedicated to the 60th Anniversary of the Pedagogical Institute of Gyumri*. Gyumri, Vysshaya Shkola, **1** (1994), 73–76 (in Russian).
6. Grigoryan E.Kh. On Solution of Problem for an Elastic Infinite Plate, One the Surface of which Finite Length Stringer is Glued. *Proc. NAS RA, Mechanics* **53** : 4 (2000), 11–16.
7. Sarkisyan V.S., Kerobyan A.V. On the Solution of the Problem for Anisotropic Half-plane on the Edge of which a Nonlinear Deformable Stringer of Finite Length is Glued. *Proc. NAS RA, Mechanics* **50** : 3–4 (1997), 17–26 (in Russian).
8. Kerobyan A.V., Sahakyan K.P. Loads Transfer from Finite Number Finite Stringers to an Elastic Half-plane Through Adhesive Shear Layers. *Proc. NAS RA, Mechanics*, **70** : 3 (2017) 39–56 (in Russian).
9. Kerobyan A.V. Transfer of Loads from a Finite Number of Elastic Overlays with Finite Lengths to an Elastic Strip Through Adhesive Shear Layers *Proc. YSU A, Phys. Math. Sci.* **53** (2019), 109–118. <https://doi.org/10.46991/PYSU:A/2019.53.2.109>
10. Aghayan K.L. Some Contact Problems for an Elastic Infinite Plate Strengthened by Elastic Overlays. *Proc. AS USSR. MSB* **5** (1972), 32–45 (in Russian).
11. Melan E. Ein Beitrag zur Theorie geschweißter Verbindungen. *Ingenieur-Archiv* **3** (1932), 123–129. <https://doi.org/10.1007/BF02079955>
12. Muki R., Sternberg E. On the Diffusion of Load From a Transverse Tension Bar Into a Semi-infinite Elastic Sheet. *J. Appl. Mech.* **4** (1968) 737–746.
<https://doi.org/10.1115/1.3601299>
13. Shilov G.E. *Mathematical Analysis, Special Course*. M. (1961), 442 p. (in Russian).

Ա. Վ. ՔԵՐՈԲՅԱՆ

ԿՊՉՈՒՆ ՍԱՆՔԻ ՇԵՐՏԵՐԻ ՄԻՋՈՑՈՎ ԵՐԿՈՒ ԶՈՒԳԱՆԵՌ ՎԵՐՋԱՎՈՐ
ԵՐԿԱՐՈՒԹՅԱՄԲ ՍՏՐԻՆԳԵՐՆԵՐՈՎ ՈՒԺԵՂԱՑՎԱԾ ԱՌԱՉԳԱԿԱՆ ԱՆՎԵՐՋ
ՍԱԼԻ ՆԱՄԱՐ ՄԻ ԽՆԴՐԻ ՄԱՍԻՆ

Դիսարկված է խնդիր առաձգական անվերջ սալի համար, որն իր վերին մակերևույթի վերջավոր փեղամասերում ուժեղացված է երկու իրար զուգահեռ,

վերջավոր երկարությամբ և փարբեր առաձգական և երկրաչափական բնութագրեր ունեցող սպրինգերներով: Սալի հորիզոնական առանցքի նկատմամբ սպրինգերները դասավորված են ոչ համաչափ և դեֆորմացիայի են ենթարկվում իրենց ծայրերում կիրառված հորիզոնական ուժերի ազդեցության փակ: Անվերջ սալի և սպրինգերների միջև փոխազդեցությունը իրագործվում է այլ ֆիզիկամեխանիկական և երկրաչափական բնութագրեր ունեցող կաշուն սահքի շերտերի միջոցով: Անհայտ շոշափող լարումների որոշման խնդիրը հանգեցված է իրար զուգահեռ վերջավոր միջակայքերում երկու անհայտ ֆունկցիաների նկատմամբ Ֆրեդհոլմի երկրորդ սեռի ինֆեգրալ հավասարման համակարգի լուծմանը: Ցույց է տրված, որ խնդրի բնութագրիչ պարամետրի փոփոխման որոշ փրոյոյթում այդ հավասարումների համակարգը Բանախի փարածության մեջ կարելի է լուծել հաջորդական մոտավորությունների մեթոդով: Դիտարկված են որոշ մասնավոր դեպքեր և պարզաբանված է զուգահեռ վերջավոր տեղամասերում գործող անհայտ շոշափող լարումների վարքը և բնույթը:

А. В. КЕРОПЯН

ОБ ОДНОЙ ЗАДАЧЕ ДЛЯ УПРУГОЙ БЕСКОНЕЧНОЙ ПЛАСТИНЫ,
УСИЛЕННОЙ ДВУМЯ ПАРАЛЛЕЛЬНЫМИ СТРИНГЕРАМИ КОНЕЧНЫХ
ДЛИН ПОСРЕДСТВОМ ЛИПКИХ СДВИГОВЫХ СЛОЕВ

Рассматривается задача упругой бесконечной пластины, которая на конечных участках своей верхней поверхности усилена двумя параллельными стрингерами конечной длины с различными модулями упругости и геометрическими характеристиками. Стрингеры расположены несимметрично относительно горизонтальной оси пластины и деформируются под действием горизонтальных сил, приложенных на ее концах. Контактные связи между пластиной и стрингерами осуществляются посредством одинаковых тонких, липких слоев с другими физико-механическими и геометрическими характеристиками. В работе задача определения закона распределения неизвестных касательных напряжений, действующих между бесконечной пластиной и стрингерами, сведена к решению системы интегральных уравнений Фредгольма второго рода с двумя неизвестными функциями, определенными на двух параллельных конечных интервалах. Показывается, что в определенной области изменения характерных параметров задачи полученная система интегральных уравнений в банаховом пространстве может быть решена методом последовательных приближений. Рассмотрены некоторые частные случаи и выяснен характер и поведение неизвестных касательных напряжений, действующих на параллельных конечных участках.