

INVESTIGATION OF THE INFLUENCE OF AN INTERMEDIATE HINGE  
SUPPORT IN THE PROBLEM OF BENDING OF AN ELASTICALLY  
RESTRAINED ORTHOTROPIC BEAM

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In this paper, based on the refined theory of orthotropic plates of variable thickness, a system of differential equations is obtained for solving the problem of bending of an elastically restrained beam with an intermediate condition. The beam thickness is constant and is subject to a uniformly distributed load. The effects of transverse shear are also taken into account. Passing to dimensionless quantities, an analytical closed solution is obtained. The question of the influence of changing the place of application of the intermediate condition on the solution is discussed. Depending on the location of the hinge bearing, the question of optimality was posed and resolved according to the principle of minimum maximum deflection. The results are presented in both tabular and graphical form. Based on the results obtained, appropriate conclusions are drawn.

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**Keywords:** elastically fastened support, bending, transverse shear, minimax, optimality.

**Introduction.** It is known that the structural elements used in different building structures are in the form of beams, plates or shells. To find out and ensure the bearing capacity of such thin-walled elements, sometimes it becomes necessary to solve the bending problem with an additional intermediate condition. Considering that modern structural elements are mainly anisotropic, the study of the stress-strain state must be carried out according to refined theories. In the present work, the problem under consideration is solved precisely according to this theory. This paper addresses the following issues:

1. Obtaining and solving a differential equation describing the bending of an orthotropic beam with the corresponding boundary conditions.
2. Numerical calculations for different values of the parameter of the elastically restrained support and with variation of the place of application of the hinged support, taking into account the transverse shear and without it.

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3. Based on the principle of minimum maximum deflection, determining the best option for the place of application of the hinge bearing.

**Problem Setting.** Consider an orthotropic beam of length  $l$ , constant width  $b$ , and constant thickness  $h$ . The main directions of the anisotropy of the material are parallel to the coordinate axes  $x, y, z$ . The beam is elastically clamped at the right end, the left edge is rigidly fastened, and at some distance from the left edge it is hingedly supported. The beam is under a uniformly distributed load (Fig. 1). The conditions of the considered elastically fastened support during transverse bending of the beam are known [1], and have the form:

$$\frac{dw}{dx} = D(aN_x - M_x), \quad w = a\frac{dw}{dx} + BN_x, \quad (1)$$

here  $w$  is the deflection;  $N_x$  and  $M_x$  are the transverse force and bending moment of the beam;  $D$  and  $B$  are the parameters of the elastically restrained support, which are related by the relation

$$D = \frac{3B}{a^2} \quad (2)$$

and are the reciprocal of the stiffness of the support for rotation and vertical displacement, respectively.

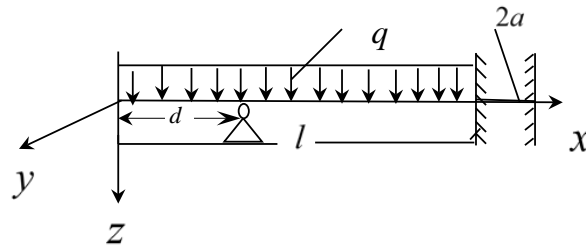


Fig. 1. The Beam center axis and load shape.

By using the refined theory of orthotropic plates of variable thickness [1], we obtain the differential equations for the problem of bending of the considered beam:

$$\begin{cases} \left( Eh^2 \frac{d^2 h}{dx^2} \right) \frac{d^2 w}{dx^2} - h \left( 8 + \chi h \frac{d^2 h}{dx^2} \right) \frac{d\varphi_1}{dx} - 16 \frac{dh}{dx} \varphi_1 = 12q, \\ Eh^2 \frac{d^3 w}{dx^3} + 2Eh \frac{dh}{dx} \cdot \frac{d^2 w}{dx^2} - \chi h^2 \frac{d^2 \varphi_1}{dx^2} - 2\chi h \frac{dh}{dx} \cdot \frac{d\varphi_1}{dx} + \varphi_1 = 0, \end{cases} \quad (3)$$

here  $E$  is Young's modulus;  $\chi$  – takes into account the effect of the transverse shear of the deformation  $e_{xz}$ ;  $\varphi_1$  is function characterizing the distribution of the shear stress  $\tau_{xz}$  on the median plane  $z = 0$  of the beam.

The boundary conditions for hinged support are as follows:

$$w = 0, \quad M_x = 0, \quad (4)$$

and for  $x = l$  in the form (1).

Consider a beam of the constant thickness  $h$ . We will apply the following substitution with the variables:

$$\begin{aligned} x = l\bar{x}, \quad h = m_1 l, \quad b = m_2 l, \quad d = m_4 l, \quad a = m_3 l, \quad \varphi_1 = E\bar{\varphi}, \quad w = h\bar{w}, \quad Ea_{55} = \chi, \\ q = E\bar{q}m_1^3, \quad D = \frac{3\bar{B}}{Em_3^2 l^3}, \quad BEl = \bar{B}, \quad N_x = Eh^2\bar{N}_x, \quad M_x = Eh^3\bar{M}_x. \end{aligned} \quad (5)$$

From the system (3) we get:

$$\begin{cases} \frac{d\bar{\varphi}}{d\bar{x}} = -\frac{3}{2}m_1^2\bar{q}, \\ \frac{m_1^3 d^3\bar{w}}{d\bar{x}^3} + 8\bar{\varphi} = 0. \end{cases} \quad (6)$$

The boundary conditions, taking into account (5), will be:

at  $\bar{x} = 0$

$$\bar{w}|_{\bar{x}=0} = 0, \quad \frac{d\bar{w}}{d\bar{x}}|_{\bar{x}=0} = 0; \quad (7)$$

at  $\bar{x} = m_4$

$$\bar{w}|_{\bar{x}=m_4} = 0, \quad \bar{M}|_{\bar{x}=m_4} = 0; \quad (8)$$

at  $\bar{x} = 1$

$$\bar{w}|_{\bar{x}=1} = m_3 \frac{d\bar{w}}{d\bar{x}}|_{\bar{x}=1} + \bar{B}m_1\bar{N}|_{\bar{x}=1} \quad (9)$$

$$\frac{d\bar{w}}{d\bar{x}}|_{\bar{x}=1} = \frac{3\bar{B}}{m_3^2} (m_1 m_3 \bar{N}_{\bar{x}} - m_1^2 \bar{M}_{\bar{x}})|_{\bar{x}=1}.$$

Having solved the system (6), we obtain:

$$\bar{\varphi} = -\frac{3}{2}m_1^2\bar{q} \cdot \bar{x} + c_1, \quad (10)$$

$$\bar{w} = \frac{\bar{q}}{2m_1}\bar{x}^4 - \frac{4}{3m_1^3}c_1\bar{x}^3 + \frac{1}{2}c_2\bar{x}^2 + c_3\bar{x} + c_4. \quad (11)$$

For  $\bar{N}_x$  and  $\bar{M}_x$  we will have

$$\bar{N}_x = m_1 m_2 \bar{q} \bar{x} + \frac{2m_2}{3m_1} c_1, \quad (12)$$

$$\bar{M}_x = -\frac{1}{2}m_2\bar{q}\bar{x}^2 + \frac{2}{3} \cdot \frac{m_2}{m_1} c_1 \bar{x} - \frac{1}{12}m_1 m_2 c_2 - \frac{1}{4}m_1^2 m_2 \chi \bar{q}. \quad (13)$$

Inserting (11), (12) and (13) into (9), we obtain a system of linear equations for the unknown coefficients  $c_1$ ,  $c_2$ ,  $c_3$  and  $c_4$ .

The system looks like:

$$\begin{cases} A_{11}c_1 + A_{12}c_2 + A_{13}c_3 + A_{14}c_4 = B_1, \\ A_{21}c_1 + A_{22}c_2 + A_{23}c_3 + A_{24}c_4 = B_2, \\ A_{31}c_1 + A_{32}c_2 + A_{33}c_3 + A_{34}c_4 = B_3, \\ A_{41}c_1 + A_{42}c_2 + A_{43}c_3 + A_{44}c_4 = B_4, \end{cases} \quad (14)$$

where

$$A_{11} = \frac{4m_3}{m_1^3} - \frac{2m_2\bar{B}}{3} - \frac{4}{3m_1^3}, \quad A_{12} = \frac{1}{2} - m_3, \quad A_{13} = 1 - m_3, \quad (15)$$

$$A_{14} = 1, \quad A_{21} = \frac{2\bar{B}m_2}{m_3^2} - \frac{2m_2\bar{B}}{m_3} - \frac{4}{m_1^3};$$

$$A_{22} = 1 - \frac{1}{4} \cdot \frac{m_1^3 m_2 \bar{B}}{m_3^2}, \quad A_{23} = 1, \quad A_{24} = 0, \quad A_{31} = \frac{4m_4^3}{3m_1^3}, \quad (16)$$

$$A_{32} = -\frac{1}{2}m_4^2, \quad A_{33} = -m_4, \quad A_{34} = -1;$$

$$A_{41} = \frac{2m_2m_4}{3m_1^2}, \quad A_{42} = -\frac{m_1m_2}{12}, \quad A_{43} = 0, \quad A_{44} = 0, \quad (17)$$

$$B_1 = \frac{2\bar{q}m_3}{m_1} - \bar{B}m_1^2m_2\bar{q} - \frac{\bar{q}}{2m_1};$$

$$B_2 = \frac{3\bar{B}m_1^4m_2\chi\bar{q}}{8m_3^2} - \frac{3\bar{B}m_1^2m_2\bar{q}}{m_3} + \frac{3\bar{B}m_1^2m_2\bar{q}}{2m_3^2} - \frac{2\bar{q}}{m_1}, \quad B_3 = \frac{\bar{q}m_4^4}{2m_1}, \quad (18)$$

$$B_4 = \frac{1}{8}m_1^2m_2\chi\bar{q} + \frac{1}{2}m_2m_4^2\bar{q}.$$

Table 1

Maximum deflection values and extremum point depending on the parameter  $m_4$  and  $\bar{B}$  at  $\chi = 0$ .

$\chi=0$		$\bar{B} = 0; \bar{x} = 0$ $\bar{B} = 1; \bar{x} = 1$		$\bar{B} = 0; \bar{x} = 0$ $\bar{B} = 5; \bar{x} = 1$		$\bar{B} = 0; \bar{x} = 0$ $\bar{B} = 10; \bar{x} = 1$	
		$x \in [0; m_4]$	$x \in [m_4; 1]$	$x \in [0; m_4]$	$x \in [m_4; 1]$	$x \in [0; m_4]$	$x \in [m_4; 1]$
		$m_4=0.1$	$\bar{w}_{\max}$	0.000065	0.421428	0.000065	0.399637
	$\bar{x}_{\max}$	0.05784	0.47844	0.05784	0.47419	0.05785	0.46744
$m_4=0.2$	$\bar{w}_{\max}$	0.001039	0.263133	0.001039	0.24968	0.001039	0.22962
	$\bar{x}_{\max}$	0.11569	0.53637	0.11569	0.53256	0.11569	0.52657
$m_4=0.3$	$\bar{w}_{\max}$	0.005264	0.154268	0.005264	0.146529	0.005264	0.135053
	$\bar{x}_{\max}$	0.17354	0.59427	0.17354	0.59077	0.17354	0.58545
$m_4=0.4$	$\bar{w}_{\max}$	0.016638	0.083251	0.016638	0.079084	0.016638	0.073116
	$\bar{x}_{\max}$	0.23139	0.65208	0.23139	0.64854	0.23138	0.64349
$m_4=0.5$	$\bar{w}_{\max}$	0.040621	0.040062	0.040621	0.037802	0.04062	0.03487
	$\bar{x}_{\max}$	0.28923	0.70965	0.28923	0.70514	0.28923	0.69931
$m_4=0.6$	$\bar{w}_{\max}$	0.084231	0.016260	0.084231	0.014725	0.084231	0.013031
	$\bar{x}_{\max}$	0.34708	0.76663	0.34708	0.75886	0.34708	0.74955
$m_4=0.7$	$\bar{w}_{\max}$	0.156049	0.004888	0.156049	0.003546	0.156049	0.002231
	$\bar{x}_{\max}$	0.404925	0.82183	0.404925	0.80446	0.40492	0.78458
$m_4=0.8$	$\bar{w}_{\max}$	0.266213	0.000684	0.266213	0.005	0.266213	0.003
	$\bar{x}_{\max}$	0.46277	0.87007	0.46277	0.850	0.46277	0.82
$m_4=0.9$	$\bar{w}_{\max}$	0.426422	0.0	0.426422	0.0	0.436422	0.0
	$\bar{x}_{\max}$	0.52062	0.90002	0.52062	0.900002	0.52062	0.90

The found value and inserting it into (10)–(13), we obtain expressions for the basic quantities.

**Computational Part.** Consider a numerical example. Let

$$m_1 = 0.1, \quad m_2 = 0.3, \quad m_3 = 0.1; \quad \chi = 0, \quad 5 \text{ and } 10; \quad \bar{B} = 0, \quad 5 \text{ and } 10;$$

$m_4 = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8$  and  $0.9$ ;  $\chi = 0, 5$  and  $10$ .

For these values of the parameters, solving the system (14), we obtain the values for  $c_1, c_2, c_3$  and  $c_4$ .

By inserting found value into expressions (11), (12) and (13), we obtain the required basic functions. The calculation results are shown in Figs. 2, 3 and Tabs. 1, 2.

Table 2

Maximum deflection values and extremum point depending on the parameter  $m_4$  and  $\bar{B}$  at  $\chi = 5$

$\chi=5$		$\bar{B} = 0 ; \bar{x} = 0$ $\bar{B} = 1 ; \bar{x} = 1$		$\bar{B} = 0 ; \bar{x} = 0$ $\bar{B} = 5 ; \bar{x} = 1$		$\bar{B} = 0 ; \bar{x} = 0$ $\bar{B} = 10 ; \bar{x} = 1$	
		$x \in [0; m_4]$	$x \in [m_4; 1]$	$x \in [0; m_4]$	$x \in [m_4; 1]$	$x \in [0; m_4]$	$x \in [m_4; 1]$
$m_4=0.1$	$\bar{w}_{\max}$	0.000340	0.442168	0.000340	0.417154	0.000340	0.380003
	$\bar{x}_{\max}$	0.06501	0.47465	0.06501	0.46988	0.06501	0.46216
$m_4=0.2$	$\bar{w}_{\max}$	0.002124	0.27951	0.002123	0.263391	0.002124	0.239226
	$\bar{x}_{\max}$	0.1247	0.53217	0.1247	0.5278	0.12470	0.52073
$m_4=0.3$	$\bar{w}_{\max}$	0.007680	0.166809	0.00768	0.156968	0.007680	0.142156
	$\bar{x}_{\max}$	0.18178	0.58956	0.18178	0.58551	0.18178	0.57904
$m_4=0.4$	$\bar{w}_{\max}$	0.020911	0.092483	0.020911	0.086775	0.020911	0.078271
	$\bar{x}_{\max}$	0.23846	0.64676	0.23846	0.6428	0.23846	0.63671
$m_4=0.5$	$\bar{w}_{\max}$	0.047276	0.046509	0.047276	0.043261	0.047276	0.038622
	$\bar{x}_{\max}$	0.29529	0.70366	0.29529	0.69923	0.29529	0.69288
$m_4=0.6$	$\bar{w}_{\max}$	0.093798	0.020421	0.093798	0.018442	0.093798	0.015858
	$\bar{x}_{\max}$	0.35233	0.76002	0.35233	0.75385	0.35233	0.74563
$m_4=0.7$	$\bar{w}_{\max}$	0.169055	0.007309	0.169055	0.005901	0.169055	0.004297
	$\bar{x}_{\max}$	0.40953	0.81545	0.40953	0.80447	0.40953	0.79058
$m_4=0.8$	$\bar{w}_{\max}$	0.283187	0.001806	0.283187	0.000794	0.283187	0.000103
	$\bar{x}_{\max}$	0.46687	0.86904	0.46687	0.84535	0.46687	0.81653
$m_4=0.9$	$\bar{w}_{\max}$	0.447892	0.000059	0.447892	0.0	0.447892	0.0
	$\bar{x}_{\max}$	0.5243	0.91324	0.5243	0.900002	0.5243	0.9

Tabs. 1 and 2 show, respectively, the values of the maximum and the place of its acceptance on the segments  $[0; m_4]$  and  $[m_4; 1]$  at  $\chi = 0, \chi = 5$  and at different values of some the basic parameters.

Now, answering the third question, we see (see Figs. 2, 3 and Tabs. 1 and 2) that the maximum value of the deflection  $\bar{x} \in [0; m_4]$  and  $\bar{x} \in [m_4; 1]$  simultaneously on two parts of the beam will be minimal at  $m_4 = 0.5$ .

In the scientific literature, you can find many works devoted to the description and application of different methods for studying the bending of Thin-Walled elements with different boundary conditions, including under the condition of an elastically restrained support [1–20]

### Conclusion.

1. The data given in Tabs. 1 and 2 show that in all cases the maximum deflection value at the intervals  $\bar{x} \in [0; m_4]$  and  $\bar{x} \in [m_4; 1]$  will not be in the middle of these segments, but at the closer to the hinge support.

2. Taking into account the effect of transverse shear deformations (cases  $\chi > 0$ ), as expected, with the same values of other quantities, leads to an increase in deflections.

3. In the case of an elastically restrained support (cases  $B > 0$ ), and with the same values of other values, it leads to an increase in deflections, compared to rigid restraint (cases  $B = 0$ ).

4. Changing parameter  $\bar{B}$  does not affect the amount of deflection in the interval  $\bar{x} \in [0; m_4]$ .

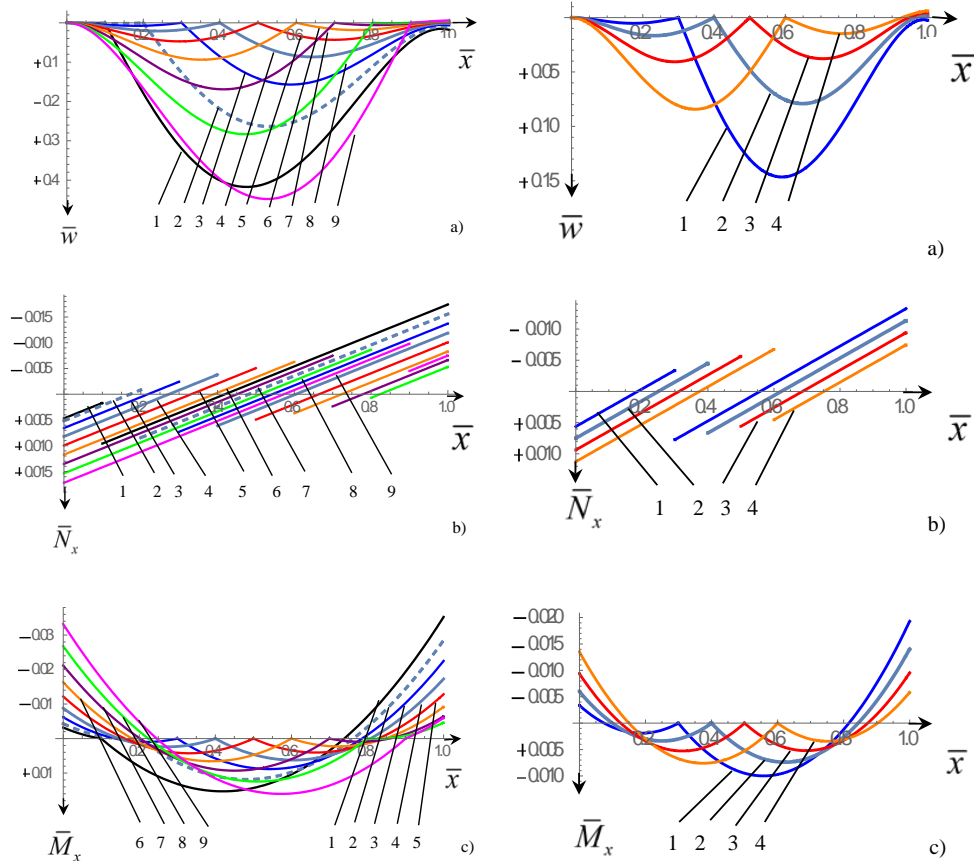


Fig. 2. Distribution of deflection (a), transverse force (b), and bending moment (c) depending on the parameter  $m_4$  at  $\bar{B} = 5, \chi = 5$ .

- 1  $\rightarrow m_4 = 0.1$ ; 2  $\rightarrow m_4 = 0.2$ ;
- 3  $\rightarrow m_4 = 0.3$ ; 4  $\rightarrow m_4 = 0.4$ ;
- 5  $\rightarrow m_4 = 0.5$ ; 6  $\rightarrow m_4 = 0.6$ ;
- 7  $\rightarrow m_4 = 0.7$ ; 8  $\rightarrow m_4 = 0.8$ ; 9  $\rightarrow m_4 = 0.9$ .

Fig. 3. Distribution of deflection (a), transverse force (b), and bending moment (c) depending on the parameter  $m_4$  at  $\bar{B} = 5, \chi = 0$ .

- 1  $\rightarrow m_4 = 0.3$ ; 2  $\rightarrow m_4 = 0.4$ ;
- 3  $\rightarrow m_4 = 0.5$ ; 4  $\rightarrow m_4 = 0.6$ .

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#### Ս. Պ. ՍՏԵՓԱՆՅԱՆ

##### ՄԻՋԱՆԿՅԱԼ ՆՈՂԱԿԱՊՈՐԵՆ ՆԵՆԱՐԱՆԻ ԱԶԴԵՑՈՒԹՅԱՆ ՆԵՏԱԶՆՈՏՈՒԹՅՈՒՆԸ ԱՌԱԶԳԱԿԱՆ-ԱՄՐԱԿՑՎԱԾ ՕՐԹՈՏՐՈՊ ՆԵԾԱՆԻ ԾՈՄԱՆ ԽՆԴՐՈՒՄ

Այս հոդվածում, փոփոխական հասարության սալերի ճշգրտված փետության հիման վրա, սրացվել է առաձգական ամրակցված և միջանկյալ պայմանով հեծանի ծռման խնդրի լուծման համար դիֆերենցիալ հավասարումների համակարգը: Նեծանի հասարությունը հասարարուն է և գրնվում է հավասարաչափ բաշխված բեռի ազդեցության փակ: Նաշվի է առնվում նաև ընդլայնական սահքի ազդեցությունը: Անցնելով անչափ մեծությունների սրացվել է անալիտիկ փակ լուծում: Քննարկվել է միջանկյալ պայմանի կիրառման փեղի փոփոխության ազդեցությունը լուծման վրա: Կախված հողակապորեն հենման փեղի փոփոխությունից, ըստ մեծագույն ճկվածքի մինիմալացման սկզբունքի դրվել և լուծվել է օպտիմալության հարցը: Արդյունքները ներկայացված են ինչպես աղյուսյակային այնպես էլ գրաֆիկական փեսքով: Սրացված արդյունքների հիման վրա կափարվում են համապարասխան հեքրություններ:

#### С. П. СТЕПАНИЯН

##### ИССЛЕДОВАНИЕ ВЛИЯНИЯ ПРОМЕЖУТОЧНОЙ ШАРНИРНОЙ ОПОРЫ В ЗАДАЧЕ ИЗГИБА УПРУГО-ЗАЩЕМЛЕННОЙ ОРТОТРОПНОЙ БАЛКИ

В этой статье на основе уточненной теории ортотропных пластин переменной толщины получена система дифференциальных уравнений для решения задачи изгиба упруго-защемленной балки с промежуточным условием. Толщина балки постоянная и находится под действием равномерно распределенной нагрузки. Учитываются также влияния поперечного сдвига. С переходом к безразмерным величинам получено аналитическое замкнутое решение. Обсуждается вопрос влияния изменения места приложения промежуточного условия на решение. В зависимости от места нахождения шарнирного опирания поставлен и решен вопрос оптимальности по принципу минимальности максимального прогиба. Результаты представлены как в табличной, так и в графической форме. На основе полученных результатов делаются соответствующие выводы.