**PROCEEDINGS OF THE YEREVAN STATE UNIVERSITY**

*Physical and Mathematical Sciences* 2021, **55**(3), p. 169–173

*Physics*

# ANALOGUE OF THE ABRAHAM–MINKOWSKI CONTROVERSY IN ELECTRONIC OPTICS

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In the problem of electron diffraction by a standing light wave (the Kapitza–Dirac effect), an electronic refractive index can be defined as the ratio of electron momenta in the wave field and outside it. Moreover, both kinetic and canonical electron momenta can be used for this purpose, which corresponds to the Abraham–Minkowski controversy in photonic optics. It is shown that in both cases the same expression for the electronic refractive index is obtained. This is consistent with Barnett's resolution of the Abraham–Minkowski dilemma.

https://doi.org/10.46991/PYSU:A/2021.55.3.169

***Keywords***: Kapitza–Dirac effect, electron optics, Abraham–Minkowski dilemma, Barnett's resolution.

**Introduction.** According to Ginzburg [1], "There are in physics a few literally 'perpetual problems' which continue to be discussed in the scientific literature for decades and decades." On all grounds, these problems include what is commonly referred to as the Abraham–Minkowski dilemma or controversy. The Abraham–Minkowski dilemma has been discussed for over a century regarding the correct form of the energy-momentum tensor of an electromagnetic field in ponderable media. Many studies [2–6] have been developed in favor of the energy-momentum tensor of the electromagnetic field in a dielectric medium both in the Abraham form [7, 8] and in favor of the expression proposed by Minkowski [9]. Moreover, other expressions have been suggested for a similar purpose [10]. However, due to the lack of irrefutable arguments in favor of one of the existing approaches, the controversy continues.

In the previous article [11], we have made a preliminary report on some observations about the momentum of an electromagnetic wave propagating in a dielectric medium with a time-varying permittivity. It was shown that the momentum of an electromagnetic wave in the form of Minkowski is preserved with an instantaneous change in the dielectric permittivity of the medium. At the same time, the Abraham

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momentum is not conserved, despite the spatial homogeneity of the problem. This circumstance was interpreted as a manifestation of the Abraham force.

Before presenting a complete account of the research on the problem of correct formulation of macroscopic electrodynamics of continuous moving media, which goes beyond merely discussing the issue of various expressions proposed for the energy-momentum tensor of an electromagnetic field in ponderable media, here we make another observation about the presence of an analogue of the Abraham–Minkowski dilemma in electronic optics.

**Refraction Index for Electrons Passing a Standing Light Wave.** In 1933, Kapitza and Dirac predicted [12] that the standing light wave can act as a diffraction grating for electrons passing through its periodic structure, similar to the periodic lattice of crystals in the experiments of Davisson and Germer [13]. The Kapitza–Dirac effect is nowadays an experimentally established phenomenon [14] and arouses a tangible interest [15, 16].

Here we are interested in the elementary approach developed in [17] to describe the Kapitza–Dirac effect in the framework of electron optics. A similar method was employed in [18] to describe an analogous effect predicted in [19] for the diffraction of an electron beam by a traveling laser wave propagating in a dielectric medium .

Thus, electrons with mass $m$ and charge $e = -|e|$ at an angle $\theta$ fall on a monochromatic linearly polarized laser beam propagating in a medium with a refractive index $n$. Choosing the axes $x$ and $y$ in the direction of propagation and polarization of the laser wave, respectively, we can represent the electromagnetic field in the following form:

$$A^{\mu}(x,t) = (0,\ \boldsymbol{A}) = (0,\ 0,\ A\cos(kx - \omega t),\ 0), \tag{1}$$

where $\mu = 0, \ldots, 3$, and wave amplitude $A$ represents the intensity of the laser beam: $I \sim n\omega^2 A^2$. The four-wavevector has the following form: $k^{\mu} = (\omega/c,\ \boldsymbol{k}) = (\omega/c,\ k,\ 0,\ 0)$, where $\omega$ and $k = \omega n/c$ represent the angular frequency and wavenumber of the wave, respectively. The four-momentum of electrons incident on a light beam at an angle $\theta$ has the following form:

$$p^{\mu}_{in} = \left(\frac{\mathcal{E}_{in}}{c},\ \boldsymbol{p}_{in}\right) = \left(\frac{\mathcal{E}_{in}}{c},\ p_{in}\sin\theta,\ p_{in}\cos\theta,\ 0\right), \tag{2}$$

where the energy $\mathcal{E}_{in}$ and the three-dimensional momentum $\boldsymbol{p}_{in}$ are related by the standard relativistic relation $\mathcal{E}^2_{in} = m^2c^4 + p^2_{in}c^2$. For simplicity, without loss of generality, we have chosen $p^{(z)}_{in} = 0$, which means that the interaction of electrons with the laser beam occurs in the $x - y$ plane (Fig.1 a).

Since light propagates in a dielectric medium at a speed $u$ less than the speed of light in vacuum: $u = c/n < c$, we can move from the laboratory coordinate system to the frame of reference moving in the direction of wave propagation at the speed $u$ of light in the medium, which we will contingently call the wave frame of reference. For the four-dimensional vector potential of the electromagnetic field of a light wave in the wave frame of reference, we have:

$$\tilde{A}^{\mu}(\tilde{x}, \tilde{t}) = (0,\ 0,\ \tilde{A}\cos(\tilde{k}\tilde{x}),\ 0), \tag{3}$$

which is actually the solution of the Maxwell and Minkowski equations for a plane electromagnetic wave "propagating" in a moving medium. In this relation, $\tilde{k} = \omega\sqrt{n^2-1}/c$ is the wave number of the laser beam in its own frame of reference, at the same time determines the spatial periodicity of the diffraction grating: $\tilde{\lambda} = 2\pi/\tilde{k}$. In addition, the diffraction grating represented by the laser beam in this frame of reference has a stationary character: $\tilde{\omega} \sim n\omega - kc = 0$.

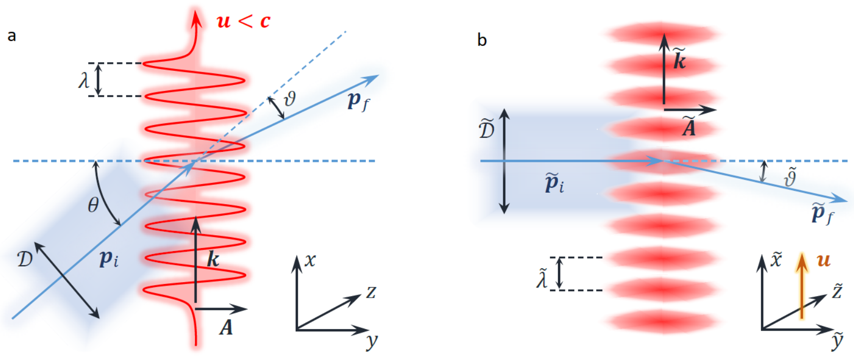


Fig. 1. Geometry of electron scattering by a laser beam in a dielectric medium in laboratory (a) and wave (b) reference frames.

For we are interested in a specific geometry, when electrons in the frame of reference associated with the laser beam fall on it at a right angle (Fig. 1 b), as in the usual Kapitza–Dirac effect in the Raman–Nath regime, it is necessary to require that in the laboratory frame of reference in the direction of light propagation the electron beam moved with the speed $u$ of propagation of the light beam: $v_{in}^{(x)} = c^2 p_{in}^{(x)}/\mathcal{E}_{in} = u$. In this case, for a four-momentum of electrons, one can obtain:

$$\tilde{p}_{in}^{\mu} = \left(\frac{\tilde{\mathcal{E}}_{in}}{c},\ 0,\ \tilde{p}_{in},\ 0\right). \tag{4}$$

We are now ready to apply Hamilton's analogy, representing the "trajectory" of electrons in an electromagnetic field as the "optical path" traveled by electrons in a medium with a variable refractive index. For this, one need to determine the refractive index of electrons moving in the laser field as the inverse ratio of the kinetic momenta of electrons in and outside the laser field: $N(\tilde{x}) \equiv \tilde{p}_{in}/\tilde{p}(\tilde{x})$. Given the stationary nature of the light wave in the wave frame associated with the light beam, we can apply the law of conservation of energy:

$$m^2c^4 + \tilde{p}_{in}^2 c^2 = m^2c^4 + \left(\tilde{\boldsymbol{p}}(\tilde{x}) - \frac{e\tilde{\boldsymbol{A}}(\tilde{x})}{c}\right)^2 c^2. \tag{5}$$

Then, taking into account relation Eq. 3 for the four-dimensional vector potential of the light wave field, for the refractive index defined above we obtain:

$$N(\tilde{x}) \approx 1 - \frac{e\tilde{A}}{c\tilde{p}_{in}} \cos(\tilde{k}\tilde{x}). \tag{6}$$

When deriving this formula, working in the Raman–Nath approximation, we neglected the angle between the vectors $\tilde{\boldsymbol{p}}$ and $\tilde{\boldsymbol{A}}$.

Further calculations within the framework of the problem of electron diffraction in the considered configuration can be found in [18]. Here we analyze the question of the electron-optical refractive index. Note that above when defining the electron-optical refractive index, we mimicked Abraham's approach defining the refractive index as the inverse ratio of the kinetic momentum of electrons in and outside the laser wave field. Meanwhile, following the Minkowski's approach, we could define the electron-optical refractive index as a direct ratio of the canonical momentum of electrons in the field of a laser wave field and outside it:

$$N(\tilde{x}) = \frac{|\,\tilde{\boldsymbol{p}}(\tilde{x}) - \frac{e}{c}\tilde{\boldsymbol{A}}\cos(\tilde{k}\tilde{x})|}{\tilde{p}_{in}} \approx 1 - \frac{e\tilde{A}}{c\tilde{p}_{in}} \cos(\tilde{k}\tilde{x}) + \dots. \tag{7}$$

Here, when obtaining the last result, we have taken into account that within the framework of the Raman–Nath approximation the kinetic energy of electrons is neglected due to the short duration of the interaction of electrons with a light wave. As a result, one can neglect the change in the kinetic momentum of electrons: $\tilde{p}(\tilde{x}) \approx \tilde{p}_{in}$.

Now we note that formulas 6 and 7 are completely identical, and the approximations used along the way to derive these formulas are consistent.

**Conclusion.** Thus, as a result of simulating Abraham's approach, we get the Eq. 6 for the electron-optical refractive index. But it is completely identical to the Eq. 7 for the electron-optical refractive index obtained by mimicking Minkowski's approach. This is essentially a confirmation of Barnett's interpretation [4] of the Abraham–Minkowski contraversy (or duality) within electron optics.

*Received 11.11.2021*
*Reviewed 08.12.2021*
*Accepted 16.12.2021*

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## ԱԲՐԱՀԱՄ–ՄԻՆԿՈՎՍԿՈՒ ՀԱԿԱՍՈՒԹՅԱՆ ՆՄԱՆԱԿԸ ԷԼԵԿՏՐՈՆԱՅԻՆ ՕՊՏԻԿԱՅՈՒՄ

Կանգնած լուսային ալիքի վրա էլեկտրոնների դիֆրակցիայի խնդրում (Կապիցա–Դիրակի էֆեկտ) կարելի է սահմանել էլեկտրոնային բեկման ցուցիչը որպես ալիքային դաշտում և դաշտից դուրս էլեկտրոնային իմպուլսների հարաբերություն: Ավելին, այդ նպատակով կարելի է օգտագործել էլեկտրոնների և՛ կինետիկ, և՛ կանոնական իմպուլսները, ինչը համապատասխանում է Աբրահամ–Մինկովսկու հակասությանը ֆոտոնային օպտիկայում: Ցույց է տրված, որ երկու դեպքում էլ էլեկտրոնային բեկման ցուցիչի համար ստացվում է նույն արտահայտությունը: Սա համահունչ է Աբրահամ–Մինկովսկու երկընտրանքի Բարնետի լուծմանը:

К. К. ГРИГОРЯН

## АНАЛОГ ПРОТИВОРЕЧИЯ АБРАХАМА–МИНКОВСКОГО В ЭЛЕКТРОННОЙ ОПТИКЕ

В задаче дифракции электронов на стоячей световой волне (эффект Капицы–Дирака) можно определить электронный показатель преломления как отношение импульсов электронов в волновом поле и вне его. Более того, для этой цели можно использовать как кинетический, так и канонический импульсы электронов, что соответствует противоречию Абрахама–Минковского в фотонной оптике. Показано, что в обоих случаях получается одно и то же выражение для электронного показателя преломления. Это согласуется с решением Барнетта дилеммы Абрахама–Минковского.