

RELATION BETWEEN THE COVARIOGRAM AND DISTRIBUTION  
FUNCTION OF THE DISTANCE BETWEEN TWO UNIFORM  
AND INDEPENDENT POINTS

V. K. OHANYAN\* , V. H. KHALATYAN \*\*

*Chair of the Theory of Probability and Mathematical Statistics*

In the present paper we obtain a relationship between the covariogram and distribution function of the distance between two uniformly and independently distributed points. Additionally, we calculate the distribution function of the distance between these two points in a disk, a ball and a triangle.

<https://doi.org/10.46991/PYSU:A/2022.56.1.033>

**MSC2020:** 60D05, 52A22, 53C65.

**Keywords:** stochastic geometry, chord length distribution function, covariogram, bounded convex body.

**Introduction.** There are many articles that aim to calculate the covariogram or the oriented dependent chord length distribution function. For example, in [1] obtained relationship between the covariogram and the orientation-dependent chord length distribution function of a cylinder and those of its base. In [2–4] authors get explicit form of the covariogram of parallelogram, triangle, right trapezoid etc.

In paper [5], using Dirac's  $\delta$ -function in Pleijel's identity, calculated the chord length distribution function for regular polygons. In [6] proved the same result for the chord length distribution function for regular polygons, using elementary geometry tools. Using this function, the author obtained the density function and the distribution function of the distance between two uniformly and independently distributed random points in the regular polygon.

The aim of the present paper is to give the relationship between distribution function of distance between two uniformly and independently chosen points and covariogram. We calculate density function of distance between two randomly and independently chosen points in a disk, ball and triangle. The density function of the distance between two points chosen independently in the ball of diameter  $d$

\* E-mail: [victoohanyan@ysu.am](mailto:victoohanyan@ysu.am),

\*\* E-mail: [v.khalatyan@ysu.am](mailto:v.khalatyan@ysu.am)

was calculated in [7]. Authors use the oriented dependent chord length distribution function, while here we use the covariogram.

**Relationship between the Covariogram and the Distribution Function of the Distance between Two Independently and Uniformly Distributed Points.**

Let  $D$  be a bounded, convex body in  $n$ -dimensional Euclidean space, with interior points. Let  $P_1$  and  $P_2$  be two points chosen at random, independently and with uniform distribution in  $D$ . Denote by  $(x_{11}, x_{12}, \dots, x_{1n})$  the coordinates of  $P_1$  and by  $(x_{21}, x_{22}, \dots, x_{2n})$  the coordinates of  $P_2$ . Denote by  $(x_1, x_2, \dots, x_n)$  difference of corresponding coordinates of  $P_1$  and  $P_2$  ( $x_i = x_{1i} - x_{2i}$ ). We define distribution function of difference of two points by

$$F_{P_1-P_2}(t) = \frac{1}{V_n^2(D)} \int_{(P_1, P_2 \in D: x_1 \leq t_1, x_2 \leq t_2, \dots, x_n \leq t_n)} dP_1 dP_2,$$

where  $t \in R^n$  and denote the coordinates of  $t = (t_1, t_2, \dots, t_n)$ . Denote density function of  $F_{P_1-P_2}(t)$  by  $f_{P_1-P_2}(t)$ .

$$\begin{aligned} f_{P_1-P_2}(t) &= \frac{d}{dt} F_{P_1-P_2}(t) \\ &= \frac{1}{V_n^2(D)} \cdot \frac{d}{dt} \left( \int_{-\infty}^{\infty} \int_{-\infty}^{t_1+x_{21}} \int_{-\infty}^{\infty} \int_{-\infty}^{t_2+x_{22}} \cdots \int_{-\infty}^{\infty} \int_{-\infty}^{t_n+x_{2n}} I_D(P_1) I_D(P_2) dP_1 dP_2 \right) \\ &= \frac{1}{V_n^2(D)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} I_D(P_2+t) I_D(P_2) dP_2. \end{aligned} \quad (1)$$

For calculating the derivative we use Leibniz integral rule, which says

$$\frac{d}{dx} \left( \int_{a(x)}^{b(x)} f(x, l) dl \right) = f(x, b(x)) \frac{d}{dx} b(x) - f(x, a(x)) \frac{d}{dx} a(x) + \int_{a(x)}^{b(x)} \frac{\partial}{\partial x} f(x, l) dl.$$

Denote  $z = \sqrt{x_1^2 + x_2^2 + \dots + x_n^2}$ . Now calculate the probability for the distance of points to be less than or equal to  $r$ .

$$\begin{aligned} F_z(r) &= P(z \leq r) = P(x_1^2 + x_2^2 + \dots + x_n^2 \leq r^2) \\ &= \int_{-r}^r \int_{-\sqrt{r^2-x_1^2}}^{\sqrt{r^2-x_1^2}} \cdots \int_{-\sqrt{r^2-x_1^2-x_2^2-\dots-x_{n-1}^2}}^{\sqrt{r^2-x_1^2-x_2^2-\dots-x_{n-1}^2}} f_{P_1-P_2}(x_1, x_2, \dots, x_n) dx_n \cdots dx_2 dx_1, \end{aligned}$$

if  $z$  is less than or equal 0, then the probability is 0. We show that  $F_z(r)$  has a connection with  $f_{P_1-P_2}(t)$ .

If we take derivative of above equation, we get the density function of two points, whose distance is less than or equal  $r$  and we should use Leibniz integral rule.

Now we can calculate the derivative of  $F_z(r)$ :

$$\begin{aligned}
f_z(r) = \frac{dF_z(r)}{dr} &= \int_{-r}^r \int_{-\sqrt{r^2-x_1^2}}^{\sqrt{r^2-x_1^2}} \cdots \int_{-\sqrt{r^2-x_1^2-x_2^2-\cdots-x_{n-2}^2}}^{\sqrt{r^2-x_1^2-x_2^2-\cdots-x_{n-2}^2}} \frac{r}{\sqrt{r^2-x_1^2-x_2^2-\cdots-x_{n-1}^2}} \\
&\quad \times \left( f_{P_1-P_2}(x_1, x_2, \dots, \sqrt{r^2-x_1^2-x_2^2-\cdots-x_{n-1}^2}) \right. \\
&\quad \left. + f_{P_1-P_2}(x_1, x_2, \dots, -\sqrt{r^2-x_1^2-x_2^2-\cdots-x_{n-1}^2}) \right) dx_{n-1} \cdots dx_2 dx_1.
\end{aligned} \tag{2}$$

The covariogram  $C(D, t)$  of  $D$  is the function [8]

$$C(D, t) = V_n(D \cap (D + t)),$$

where  $t \in R^n$ ,  $V_n$  is the  $n$ -dimensional Lebesgue measure on  $R^n$  and  $D + t = \{y + t, y \in D\}$  is the translation of  $D$  by  $t$ . This functional, was introduced by Matheron [9]. The covariogram  $C(D, t)$  is invariant with respect to translation or a reflection of  $D$  (the term reflection will always mean a reflection in a point). Matheron in 1986 asked the following question:

**Covariogram Problem.** Does the covariogram determine a convex body, among all convex bodies, up to translations and reflections?

We have that

$$C(D, t) = \int_{R^n} I_D(P) I_{D+t}(P) dP. \tag{3}$$

For (1) and (3) we get that the density function of  $P_1 - P_2$  is

$$f_{P_1-P_2}(t) = \frac{1}{V_n^2(D)} \int_{R^n} I_D(P) I_D(P-t) dP.$$

If  $P - t \in D$ , then we obtain  $P \in D + t$ . This means that the previous equation can be written in the following form

$$f_{P_1-P_2}(t) = \frac{1}{V_n^2(D)} \int_{R^n} I_D(P) I_{D+t}(P) dP. \tag{4}$$

The integral part of Eq. (4) is  $C(D, t)$ , and finally

$$f_{P_1-P_2}(t) = \frac{C(D, t)}{V_n^2(D)}. \tag{5}$$

We know that [10]

$$-\frac{\partial C(D, u, h)}{\partial t} = (1 - F(u, h)) b_D(u), \tag{6}$$

where  $h > 0$ ,  $u \in S^{n-1}$  (where  $S^{n-1}$  denotes the  $(n-1)$ -dimensional unit sphere in  $R^n$  centered at the origin) and  $b_D(u)$  is the brightness function of a convex body in  $R^n$  that gives the  $(n-1)$  dimensional volume of its orthogonal projection onto hyperplane with norm  $u$ .

Every vector from  $R^n$  can be determined by its length and direction. Assume that the direction of a vector  $t$  is  $u$  and the length is  $h$ . If we integrate both parts of Eq. (6) from 0 to  $h$ , we get

$$C(D, u, h) = V_n(D) - \int_0^h (1 - F(u, s)) b_D(u) ds. \quad (7)$$

From Eqs. (5) and (7) we can obtain relationship between the density function of  $P_1 - P_2$  and the orientation depend chord length distribution function

$$f_{P_1 - P_2}(t) = \frac{1}{V_n^2(D)} \left( V_n(D) - \int_0^h (1 - F(u, s)) b_D(u) ds \right).$$

We know that

$$\begin{aligned} \int_{-r}^r \int_{-\sqrt{r^2 - x_1^2}}^{\sqrt{r^2 - x_1^2}} \cdots \int_{-\sqrt{r^2 - x_1^2 - x_2^2 - \cdots - x_{n-2}^2}}^{\sqrt{r^2 - x_1^2 - x_2^2 - \cdots - x_{n-2}^2}} \frac{r}{\sqrt{r^2 - x_1^2 - x_2^2 - \cdots - x_{n-1}^2}} dx_{n-1} \cdots dx_2 dx_1 \\ = \frac{1}{2} \int_{S_{n-1}} r^{n-1} du. \end{aligned} \quad (8)$$

From Eqs. (2), (5) and (8) we get

$$f_z(h) = \frac{1}{2} \int_{S_{n-1}} r^{n-1} \left( \frac{C(D, u, h)}{V_n^2(D)} + \frac{C(D, u, h)}{V_n^2(D)} \right) du = \frac{h^{n-1}}{V_n^2(D)} \int_{S_{n-1}} C(D, u, h) du. \quad (9)$$

We can calculate the kinematic measure, using the covariogram. We know from [7]

$$f_z(h) = \frac{h^{n-1} K(D, h)}{V_n^2(D)},$$

where  $K(D, t)$  is the kinematic measure of all oriented segments of length  $r$  that lie entirely inside  $D$  (see [10]).

From Eq. (9) and above equation we can obtain  $K(D, x)$  as follow

$$K(D, h) = \int_{S_{n-1}} C(D, u, h) du.$$

Therefore, we obtain a relationship between covariogram  $C(D, u, h)$  and the kinematic measure  $K(D, t)$ . The same result is shown in [11].

**The Case of a Disk.** In the case of disk  $D = B_d$  of diameter  $d$  the distribution function has the form

$$F_{B_d}(y) = \begin{cases} 0, & \text{if } y \leq 0, \\ 1 - \sqrt{1 - \frac{y^2}{d^2}}, & \text{if } 0 \leq y \leq d, \\ 1, & \text{if } y \geq d. \end{cases}$$

Then  $b_{C_d}(u) = d$ . Apply (6) and integrate the equation from 0 to  $x$ , we get

$$\int_0^x -\frac{\partial C(B_d, u, h)}{\partial t} dt = \int_0^x d\sqrt{1 - \frac{h^2}{d^2}} dt,$$

$$C(B_d, u, 0) - C(B_d, u, x) = \frac{d^2}{2} \arcsin \frac{x}{d} + \frac{xd}{2} \sqrt{1 - \frac{x^2}{d^2}}.$$

$C(B_d, u0) = \frac{\pi d^2}{4}$  and the final result of  $C(B_d, ux)$  is

$$C(B_d, ux) = \frac{\pi d^2}{4} - \frac{d^2}{2} \arcsin \frac{x}{d} - \frac{xd}{2} \sqrt{1 - \frac{x^2}{d^2}}. \quad (10)$$

Because (10) doesn't dependent on  $u$ , it follows that  $C(B_d, u, h) = C(B_d, \cdot, h)$ .

We know that  $V_2^2(B_d) = \frac{\pi^2 d^4}{16}$ . Using (5), we get

$$f_{P_1-P_2}(t) = \frac{C(B_d, \cdot, h)}{V_n^2(B_d)} = \frac{4}{\pi d^2} - \frac{8}{\pi^2 d^2} \arcsin \frac{h}{d} - \frac{8h}{\pi^2 d^3} \sqrt{1 - \frac{h^2}{d^2}}. \quad (11)$$

Using (2) and that disk is origin symmetric, we get

$$f_{P_1-P_2}(\sqrt{r^2 - y^2}, y) = f_{P_1-P_2}(-\sqrt{r^2 - y^2}, y).$$

$$f_z(h) = \frac{dF_z(h)}{dh} = \int_{-h}^h \frac{2h}{\sqrt{h^2 - y^2}} \left( f_{P_1-P_2}(\sqrt{h^2 - y^2}, y) \right) dy.$$

Using (10), (11) and that  $\sqrt{r^2 - y^2} = x$ , we can calculate  $f_z(t)$  for the disk.

$$\begin{aligned} f_z(h) &= \left( \frac{8}{\pi d^2} - \frac{16}{\pi^2 d^2} \arcsin \frac{h}{d} - \frac{16h}{\pi^2 d^3} \sqrt{1 - \frac{h^2}{d^2}} \right) \int_{-h}^h \frac{h}{\sqrt{h^2 - y^2}} dy \\ &= \frac{8h}{d^2} - \frac{16h}{\pi d^2} \arcsin \frac{h}{d} - \frac{16h^2}{\pi d^3} \sqrt{1 - \frac{h^2}{d^2}}. \end{aligned}$$

This is the same as in [7].

### The Case of a Ball.

$$P(z \leq r) = P(x^2 + y^2 + z^2 \leq r^2) = \int_{-r}^r \int_{-\sqrt{r^2 - x^2}}^{\sqrt{r^2 - x^2}} \int_{-\sqrt{r^2 - x^2 - y^2}}^{\sqrt{r^2 - x^2 - y^2}} f_{P_1-P_2}(x, y, z) dx dy dz.$$

Using Leibniz integral rule, we have

$$\begin{aligned} f_z(r) = \frac{dF_z(r)}{dr} &= \int_{-r}^r \int_{-\sqrt{r^2 - x^2}}^{\sqrt{r^2 - x^2}} \frac{r}{\sqrt{r^2 - x^2 - y^2}} \left( f_{P_1-P_2}(\sqrt{r^2 - x^2 - y^2}, y, z) \right. \\ &\quad \left. + f_{P_1-P_2}(-\sqrt{r^2 - x^2 - y^2}, y, z) \right) dx dy. \end{aligned} \quad (12)$$

In the case of the ball  $D = B_d$  of diameter  $d$  the distribution function is of the form

$$F_{B_d}(y) = \begin{cases} 0, & \text{if } y \leq 0, \\ \frac{y^2}{d^2}, & \text{if } 0 \leq y \leq d, \\ 1, & \text{if } y \geq d. \end{cases}$$

Using (6) and integrating the equation from 0 to  $t$ , we obtain:

$$\int_0^x -\frac{\partial C(B_d, u, h)}{\partial t} dt = \int_0^x \left(1 - \frac{y^2}{d^2}\right) \frac{\pi d^2}{4} dy,$$

$$C(B_d, u, 0) - C(B_d, u, x) = \frac{d^2}{2} \arcsin \frac{x}{d} + \frac{xd}{2} \sqrt{1 - \frac{x^2}{d^2}}.$$

$$C(B_d, u, x) = \frac{\pi d^3}{6} + \frac{\pi x^3}{12} - \frac{\pi d^2 x}{4}. \quad (13)$$

Since (13) is independent of  $u$ , we get  $C(B_d, u, h) = C(B_d, \cdot, h)$ . We know

$$\frac{C(B_d, \cdot, h)}{V_3^2(B_d)} = \frac{6}{\pi d^3} + \frac{3h^3}{\pi d^6} - \frac{9h}{\pi d^4},$$

$$f_z(h) = \frac{dF_z(h)}{dh} = \left( \frac{6}{\pi d^3} + \frac{3h^3}{\pi d^6} - \frac{9h}{\pi d^4} \right) \int_{-h}^h \int_{-\sqrt{h^2-x^2}}^{\sqrt{h^2-x^2}} \frac{2h}{\sqrt{h^2-x^2-y^2}} dy dx$$

$$= \frac{24h^2}{d^3} + \frac{12h^5}{d^6} - \frac{36h^3}{d^4}.$$

### Distribution of the Distance between Two Random Points in a Triangle.

From Eq. (9) we show, that if we have the covariogram of a body, we can calculate the distribution of the distance between two random points. Now we calculate the distribution function of the distance between two random points for a triangle.

Assume that we have an ABC triangle. We denote  $|AB| = a$ ,  $\angle CAB = \alpha$ ,  $\angle ABC = \beta$ , the area of  $\Delta ABC$  by  $S$ . Let us consider the direction of the AB ray as zero direction and the counterclockwise orientation as a positive orientation.

From [11] we know that covariogram of triangles is equal to

$$C(\Delta, u, h) = \begin{cases} \frac{(a \sin \beta - h \sin(u + \beta))^2 \sin \alpha}{2 \sin \beta \sin(\alpha + \beta)}, & u \in [0, \alpha], h \in [0, \frac{a \sin \beta}{\sin(u + \beta)}], \\ \frac{(a \sin \alpha \sin \beta - h \sin u \sin(\alpha + \beta))^2}{2 \sin \alpha \sin \beta \sin(\alpha + \beta)}, & u \in [\alpha, \pi - \beta], h \in [0, \frac{a \sin \alpha \sin \beta}{\sin(\alpha + \beta) \sin u}], \\ \frac{(a \sin \alpha - h \sin(u - \alpha))^2 \sin \beta}{2 \sin \alpha \sin(\alpha + \beta)}, & u \in [\pi - \beta, \pi], h \in [0, \frac{a \sin \alpha}{\sin(u - \alpha)}], \\ \frac{(a \sin \beta + h \sin(u + \beta))^2 \sin \alpha}{2 \sin \beta \sin(\alpha + \beta)}, & u \in [\pi, \pi + \alpha], h \in [0, -\frac{a \sin \beta}{\sin(u + \beta)}], \\ \frac{(a \sin \alpha \sin \beta + h \sin u \sin(\alpha + \beta))^2}{2 \sin \alpha \sin \beta \sin(\alpha + \beta)}, & u \in [\pi + \alpha, 2\pi - \beta], h \in [0, -\frac{a \sin \alpha \sin \beta}{\sin(\alpha + \beta) \sin u}], \\ \frac{(a \sin \alpha + h \sin(u - \alpha))^2 \sin \beta}{2 \sin \alpha \sin(\alpha + \beta)}, & u \in [2\pi - \beta, 2\pi], h \in [0, -\frac{a \sin \alpha}{\sin(u - \alpha)}], \end{cases}$$

this is an explicit form of the covariogram for any triangle.

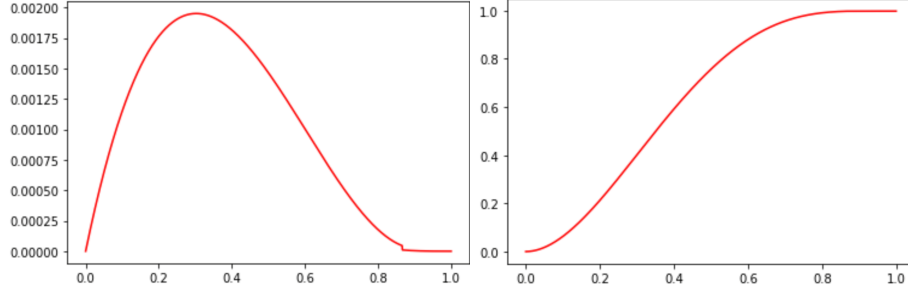
Using the Eq. (9) and the explicit form of the covariogram for triangle, we can calculate the distance between two random point in triangle.

$$\begin{aligned}
f_z(h) &= \frac{h}{S^2} \int_0^{2\pi} C(\Delta, u, h) du = \frac{h}{S^2} \left( \int_{\{u \in [0, \alpha]: \frac{a \sin \beta}{\sin(u+\beta)} \leq h\}} \frac{(a \sin \beta - h \sin(u+\beta))^2 \sin \alpha}{2 \sin \beta \sin(\alpha + \beta)} du \right) \\
&+ \int_{\{u \in [\alpha, \pi - \beta]: \frac{a \sin \alpha \sin \beta}{\sin(\alpha + \beta) \sin u} \leq h\}} \frac{(a \sin \alpha \sin \beta - h \sin u \sin(\alpha + \beta))^2}{2 \sin \alpha \sin \beta \sin(\alpha + \beta)} du \\
&+ \int_{\{u \in [\pi - \beta, \pi]: \frac{a \sin \alpha}{\sin(u - \alpha)} \leq h\}} \frac{(a \sin \alpha - h \sin(u - \alpha))^2 \sin \beta}{2 \sin \alpha \sin(\alpha + \beta)} du \\
&+ \int_{\{u \in [\pi, \pi + \alpha]: -\frac{a \sin \beta}{\sin(u + \beta)} \leq h\}} \frac{(a \sin \beta + h \sin(u + \beta))^2 \sin \alpha}{2 \sin \beta \sin(\alpha + \beta)} du \\
&+ \int_{\{u \in [\pi + \alpha, 2\pi - \beta]: -\frac{a \sin \alpha \sin \beta}{\sin(\alpha + \beta) \sin u} \leq h\}} \frac{(a \sin \alpha \sin \beta + h \sin u \sin(\alpha + \beta))^2}{2 \sin \alpha \sin \beta \sin(\alpha + \beta)} du \\
&+ \int_{\{u \in [2\pi - \beta, 2\pi]: -\frac{a \sin \alpha}{\sin(u - \alpha)} \leq h\}} \frac{(a \sin \alpha + h \sin(u - \alpha))^2 \sin \beta}{2 \sin \alpha \sin(\alpha + \beta)} du.
\end{aligned} \tag{14}$$

Assume that  $\Delta$  is an equilateral triangle whose sides are equal to  $a$ . The covariogram for the equilateral triangle is

$$C(\Delta_{eq}, u, h) = \begin{cases} \frac{\left(a \sin \frac{\pi}{3} - h \sin\left(u + \frac{\pi}{3}\right)\right)^2}{\sqrt{3}}, & u \in \left[0, \frac{\pi}{3}\right], h \in \left[0, \frac{a \sin \frac{\pi}{3}}{\sin\left(u + \frac{\pi}{3}\right)}\right], \\ \frac{\left(a \sin \frac{\pi}{3} - h \sin u\right)^2}{\sqrt{3}}, & u \in \left[\frac{\pi}{3}, \pi - \frac{\pi}{3}\right], h \in \left[0, \frac{a \sin \frac{\pi}{3}}{\sin u}\right], \\ \frac{\left(a \sin \frac{\pi}{3} - h \sin\left(u - \frac{\pi}{3}\right)\right)^2}{\sqrt{3}}, & u \in \left[\pi - \frac{\pi}{3}, \pi\right], h \in \left[0, \frac{a \sin \frac{\pi}{3}}{\sin\left(u - \frac{\pi}{3}\right)}\right], \\ \frac{\left(a \sin \frac{\pi}{3} + h \sin\left(u + \frac{\pi}{3}\right)\right)^2}{\sqrt{3}}, & u \in \left[\pi, \pi + \frac{\pi}{3}\right], h \in \left[0, -\frac{a \sin \frac{\pi}{3}}{\sin\left(u + \frac{\pi}{3}\right)}\right], \\ \frac{\left(a \sin \frac{\pi}{3} + h \sin u\right)^2}{\sqrt{3}}, & u \in \left[\pi + \frac{\pi}{3}, 2\pi - \frac{\pi}{3}\right], h \in \left[0, -\frac{a \sin \frac{\pi}{3}}{\sin u}\right], \\ \frac{\left(a \sin \frac{\pi}{3} + h \sin\left(u - \frac{\pi}{3}\right)\right)^2}{\sqrt{3}}, & u \in \left[2\pi - \frac{\pi}{3}, 2\pi\right], h \in \left[0, -\frac{a \sin \frac{\pi}{3}}{\sin\left(u - \frac{\pi}{3}\right)}\right]. \end{cases} \tag{15}$$

For calculation of two points distribution function in triangle we consider two cases.



The distribution and the density function of the distance between two uniformly and independently distributed points in equilateral triangle.

**Case 1.** Assume that  $t$  is less than or equal  $\frac{a\sqrt{3}}{2}$ . Using Eq. (14) and (15), it is obvious that 6 integrals are equal and we can calculate one integral and multiply it by 6. We know that  $S = \frac{a^2\sqrt{3}}{4}$ ,

$$\begin{aligned} f_z(h) &= \frac{h}{S^2} \int_0^{2\pi} C(\Delta_{eq}, u, h) du = \frac{6h}{S^2} \left( \int_0^{\pi/3} \frac{\left( a \sin \frac{\pi}{3} - h \sin \left( u + \frac{\pi}{3} \right) \right)^2}{\sqrt{3}} du \right) \\ &= \left( \frac{\pi a^2}{4\sqrt{3}} - ah + \frac{(2\pi + 3\sqrt{3})h^2}{12\sqrt{3}} \right) \frac{32h}{a^4}. \end{aligned}$$

**Case 2.** Assuming that  $t \in \left[ \frac{a\sqrt{3}}{2}, a \right]$ , we can do the same as in the previous case:

$$\begin{aligned} f_z(h) &= \frac{h}{S^2} \int_0^{2\pi} C(D, u, h) du = \frac{6h}{S^2} \left( \int_0^{\pi/6 - \arccos \frac{a\sqrt{3}}{2h}} \frac{\left( a \sin \frac{\pi}{3} - h \sin \left( u + \frac{\pi}{3} \right) \right)^2}{\sqrt{3}} du \right. \\ &\quad \left. + \int_{\pi/6 + \arccos \frac{a\sqrt{3}}{2h}}^{\pi/3} \frac{\left( a \sin \frac{\pi}{3} - h \sin \left( u + \frac{\pi}{3} \right) \right)^2}{\sqrt{3}} du \right) \\ &= \frac{12h}{S^2\sqrt{3}} \left( \frac{3a^2}{4} \left( \pi/6 - \arccos \frac{a\sqrt{3}}{2h} \right) - \sqrt{3}ah \left( \frac{1}{2} + \sqrt{1 - \frac{3a^2}{4h^2}} \right) \right. \\ &\quad \left. + \frac{h^2}{2} \left( -\arccos \frac{\sqrt{3}a}{2h} + \frac{\pi}{6} + \frac{1}{2} \left( \sin 2 \arccos \frac{\sqrt{3}a}{2h} - \frac{\sqrt{3}}{2} \right) \right) \right). \end{aligned}$$



**Moments of Distance between Two Points in Body from  $R^n$ .** It is shown in [7] how to calculate the  $k$ -th moment between two points randomly and independently distributed in the bounded convex domain, where it is used the chord length distribution function. We obtain the explicit form of  $k$ -th moment, using covariogram. To find the  $k$ -th moment between points (we denote it by  $M_n^k$ , where  $n$  is the dimension of the space), we need to calculate the following integral

$$M_n^k = \int_0^d x^k f_z^n(x) dx.$$

Using (9), we rewrite the last equation in the following form:

$$M_n^k = \int_0^d \frac{x^{n+k-1}}{V^2(D)} \int_{S^{n-1}} C(D, ux) dx, \quad (16)$$

using Eq. (16) we can calculate the  $k$ -th moment between two points randomly and independently distributed on any geometric object, which covariogram is known. For example, using (10) and (16) we can calculate the mean distance between two points randomly and independently distributed on the disk in the following way:

$$\begin{aligned} M_2^1 &= \int_0^d \frac{x^2}{V^2(D)} \int_0^{2\pi} \frac{\pi d^2}{4} - \frac{d^2}{2} \arcsin \frac{x}{d} - \frac{xd}{2} \sqrt{1 - \frac{x^2}{d^2}} d u dx \\ &= \frac{32}{\pi d^4} \left( \frac{\pi d^5}{12} - \frac{(3\pi - 4)d^5}{36} - \frac{d^5}{15} \right) = \frac{64}{45} d. \end{aligned}$$

This is the same result as in [7]

*The research of the present paper is partially supported by the Mathematical Studies Center at Yerevan State University.*

Received 28.02.2022

Reviewed 14.03.2022

Accepted 28.03.2022

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Վ. Կ. ՕՆՏՆՅԱՆ, Վ. Ն. ԽԱԼԱՏՅԱՆ

ԿՈՎԱՐԻՈԳՐԱՄԻ ԵՎ ՆԱՎԱՍԱՐՎՉԱՓ ԵՎ ԱՆԿԱԽ ԲԱՇԽՎԱԾ ԵՐԿՈՒ ԿԵՏԵՐԻ ՄԻՋԵՎ ՆԵՌԱՎՈՐՈՒԹՅԱՆ ԲԱՇԽՄԱՆ ՖՈՒՆԿՑԻԱՅԻ ՄԻՋԵՎ ԿԱՊ

Նոդվածում սրացվել է կապ կովարիոգրամի և հավասարաչափ և անկախ բաշխված երկու կետերի միջև հեռավորության բաշխման ֆունկցիայի միջև: Բացի դրանից, հաշվարկվել է երկու կետերի միջև հեռավորության բաշխման ֆունկցիանները շրջանի, գնդի և եռանկյան համար:

В. К. ОГАНЯН, В. А. ХАЛАТЯН

СВЯЗЬ МЕЖДУ КОВАРИОГРАММОЙ И ФУНКЦИЕЙ РАСПРЕДЕЛЕНИЯ РАССТОЯНИЯ МЕЖДУ ДВУМЯ НЕЗАВИСИМЫМИ И РАВНОМЕРНО РАСПРЕДЕЛЕННЫМИ ТОЧКАМИ

В статье получена связь между ковариограммой и функцией распределения расстояния между двумя независимыми точками с равномерным распределением в выпуклом теле. Кроме того, мы рассчитали функции распределения расстояний между такими двумя точками для круга, шара и треугольника.